Fish Harvesting Revisited

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Script for fish logistic-growth harvesting project in Section 3.5.2.

The Data: The data from Table 3.10, starting with 1978, which we will call year 0. Note Matlab indexes arrays starting with index 1, however.

```
udata = [72148, 73793, 74082, 92912, 82323, 59073, 59920, 48789, 70638, 67462, 68702, 61191, 49
```

Harvest rates from Table 3.10.

```
hd = [0.18847, 0.149741, 0.21921, 0.17678, 0.28203, 0.34528, 0.20655, 0.33819, 0.14724, 0.19757
```

Plot of the population data, indexed as years 0 to 29:

```
plot([0:29],udata)
```

And a plot of the harvest rates

```
plot([0:29],hd)
```

Fitting the Data to the ODE: Form finite difference approximations (u(k+1) - u(k))/1 from the population data, for k = 1 to k = 29). This approximates u'(t) when t = k:

```
udiffdata = udata(2:30)-udata(1:29);
```

Next substitute u = udata[k] into the harvested logistic ODE u'(t) = r*u(t)*(1-u(t)/K) - h(t)*u(t) for k = 0 to k = n-1, to approximate the right side of the ODE at time t = k for k = 0 to k = n-1. This expression depends on r and K.

```
syms r;
syms K;
u1 = udata(1:29);
udiffdata2 = r*u1.*(1-u1/K)-hd(1:29).*u1;
```

Form a sum of squares that depends on r and K

```
SS(r,K) = sum((udiffdata-udiffdata2).^2);
```

Finding the optimal r and K: Start with a plot of SS(r, K), or log(SS).

```
fsurf(log(SS), [0.1 0.5 40000 300000])
```

Or perhaps a contour plot, with contours in steps of 0.05 from 21.5 to 24:

```
fcontour(log(SS), [0.1 0.5 40000 300000],'LevelList',[21.5:0.05:24])
```

Something near r = 0.3, K = 200000 might be a good initial guess at a minimizer for SS.

If we use these values for r and K (but you should find the true minimizer) we could compare the predicted cod population to the data by following the suggestion of Modeling Exercise 5.2.3. Let U be an array to hold our numerical estimates of the cod population:

```
U = zeros(1,30);
U(1) = udata(1);
```

Now use the harvested logistic ODE with our estimates for r and K to march the predicted cod biomass out in time (this is essentially Euler's method):

```
r = 0.3;
K = 200000;
for k=1:29
    U(k+1) = U(k) + r*U(k)*(1-U(k)/K) - hd(k)*U(k);
end
```

Plot the resulting predicted population along with the data

```
plot([0:29],udata)
hold on
plot([0:29],U,'-r')
hold off
```