Chapter 6 Probability Distributions

Working with the Normal Distribution on the TI-83

Suppose IQs are distributed normally with a mean of 100 and a standard deviation of 15.

Find the proportion of people that have IQs between 80 and 120.

Step 1: The Distribution Menu. To go to the distribution menu press [2nd] [VARS]. Toggle down to 2: normalcdf and press [ENTER].

Step 2: Entering the Parameters. The order for entering the parameters are lower bound, upper bound, mean, and standard deviation---all separated by commas. Since we want to find the proportion of IQs between 80 and 120, these are our lower and upper bounds. Our mean is 100 and the standard deviation is 15. Enter these numbers as shown and press [ENTER].

Find the proportion of people that have IQs above 120.

Entering the Parameters. Since we do not have an upper bound here, we need to enter some very high number (one that is “off the chart”) for our upper bound and continue as before. I chose to enter 9999 for our upper bound. [Hint: To bring back the last thing you did on the home screen so it can be edited, press [2nd] [ENTER].]

Find what IQ you need to be in the top 5%.

Step 1: The Distribution Menu. Go to the distribution menu by pressing [2nd] [VARS]. Toggle down to 3: invNorm and press [ENTER].

Step 2: Entering the Parameters. The order for entering the parameters are proportion below the number you want, mean, and standard deviation---all separated by commas. Since we want to know what score you would need to be in the top 5%, we want to have a score that is higher than the lower 95%. Therefore, we will enter 0.95, 100, 15) and press [ENTER].
Normal Distribution Questions

1. The lengths of newborn children in the United States vary according to the normal distribution with mean 19.5 inches and standard deviation 0.75 inches.

   a) What proportion of newborns have lengths between 18 and 21 inches?

   b) What proportion of newborns are less than 18 inches in length?

   c) What proportion of newborns are more than 20 inches in length?

   d) To be in the highest 1% of lengths, above what length would a baby have to be?

Working with the Binomial Distributions on the TI-83

Suppose we are taking a five question multiple-choice test and each question has four possible answers. If a large group of people are just guessing at the answers, their results should form a binomial distribution with \( n = 5 \) and \( p = 0.25 \).

If you are guessing on our multiple choice test, find the probability that you would get exactly 2 questions right.

Step 1: The Distribution Menu. To go to the distribution menu press \([2^\text{nd}][\text{VARS}]\). Toggle down to \( \text{\textbf{0:binompdf}}( \) and press \( \text{\textbf{ENTER}} \).

Step 2: Entering the Parameters. The order for entering the parameters is \( n \), \( p \), and \( x \) (or number of repeated trials, probability of success, number of successes). In our case, \( n = 5 \), \( p = 0.25 \), and \( x = 2 \). Enter these numbers as shown and press \( \text{\textbf{ENTER}} \).
If you are guessing on our multiple choice test, find the probability that you would get 2 or fewer questions right.

**The cumulative distribution.** In this case we have \( n = 5 \) and \( p = 0.25 \) as before, but now \( x \leq 2 \). We could repeat what we did before for \( x = 2, x = 1, \) and \( x = 0 \) and add up our three answers. There is, however, an easier way. Using the \texttt{A:binomcdf(} option on our distributions menu (\texttt{2nd}[\texttt{VARS}]) we will get our answer in one step for \( x \leq 2 \). This is the binomial cumulative distribution function.

If you are guessing on our multiple choice test, find the probability that you would get more than 2 questions right.

**Area above.** The cumulative distribution function always finds the probability of a number and below. For the probability of more than 2 questions correct we need to find the probability of 2 or fewer and subtract that from one. This is just one minus our answer from the previous question.

**Binomial Distribution Formulas**

- For binomial probabilities, \( n \) = the number of repeated independent observations, 
  \( p \) = the probability of success, and \( x \) = the number of successes. The probability of \( x \) successes is:

  \[
P(x) = \binom{n}{x} p^x (1-p)^{n-x}
  \]

  where the binomial coefficient is \( \binom{n}{x} = \frac{n!}{x!(n-x)!} \).

- The mean and standard deviation of a binomial distribution are:

  \[
  \mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)}.
  \]
Binomial Distribution Questions

According to the Census Bureau, 32% of Americans bicycle. Suppose this is true. Suppose a random sample of 20 Americans is taken.

2. What is the probability that exactly 5 of them bicycle?

3. What is the probability that fewer than 5 of them bicycle?

4. What is the probability that more than 5 of them bicycle?

5. What are the mean and the standard deviation for the number of Americans that bicycle in samples of size 20?

6. Suppose a sample of 1000 Americans is taken, what is the probability that at least 300 of them bicycle?
Working with the Sampling Distributions on the TI-83

Suppose IQs are distributed normally with a mean of 100 and a standard deviation of 15.

Find the probability that a random sample of 10 people have a mean IQ greater than 110.

Step 1: The Distribution Menu. To go to the distribution menu press \[ \text{2nd} \text{[VARS]} \]. Toggle down to \[ 2: \text{normalcdf} \] and press \[ \text{ENTER} \].

Step 2: Entering the Parameters. The order for entering the parameters are lower bound, upper bound, mean, and standard deviation—all separated by commas. Since we want to find the probability that the mean IQ is above 110, our lower bound is 110 and our upper bound is infinity (we will enter some very large number). Our mean is 100 and the standard deviation is \( \frac{15}{\sqrt{10}} \). Enter these numbers as shown and press \[ \text{ENTER} \].

Sampling Distribution Questions

7. The lengths of newborn children in the United States vary according to the normal distribution with mean 19.5 inches and standard deviation 0.75 inches.

   a) What is the probability that a randomly selected newborn is more than 20 inches in length?

   b) If a random sample of 20 newborns is taken, what is the probability that the mean length is more than 20 inches?
8. Suppose the mean loss from fires for homeowners is $\mu = $250 and the standard deviation of the loss is $\sigma = $1000. This distribution is strongly skewed to the right. Many homeowners will have no loss and a few will have very large losses.

a) When we look at the distribution of mean loss from fire for a very large number of homeowners, why can we assume this distribution is approximately normal?

b) Suppose an insurance company insures 10,000 homes for losses due to fires. What is the approximate probability that the mean loss will be greater than $275?

9. A study of rush-hour traffic in San Francisco counts the number of people in each car entering a freeway at a suburban interchange. Suppose that this count has a mean of $\mu = 1.5$ and a standard deviation of $\sigma = 0.75$ in the population of all cars that enter at this interchange during rush hours.

a) Could the exact distribution of the count be normal? Why or why not?

b) Traffic engineers estimate that the capacity of the interchange is 700 cars per hour. According to the central limit theorem, what is the approximate distribution of the mean number of persons, $\bar{x}$, in 700 randomly selected cars at this interchange?

c) What is the probability that 700 cars will carry more than 1075 people?