Solutions for Section 3.1 Reading Questions

1. A tax credit is the direct reduction in the amount of tax one has to pay. It is a change in the output because tax credit is the amount of tax owed, which is an output.

2. Adding a constant to a function shifts the function's graph vertically because adding a constant affects the output of a function which is shown on the vertical axis.

3. $g(x) = f(x) - 1 = 2x - 3 - 1 = 2x - 4$. The graph of g is the graph of f shifted down by 1 unit.

4. When the function is shifted vertically, all the points are shifted the same amount. However, when the function is stretched vertically, all the points are moved away from (or towards) the x-axis by a constant factor.

5. $f(x) = 2g(x)$ or $g(x) = \frac{1}{2}f(x)$.

6. All of the points of $y = a \cdot f(x)$ are closer to the x-axis than the corresponding points of $y = b \cdot f(x)$.

7. All the points on the x-axis do not change when the graph is stretched.

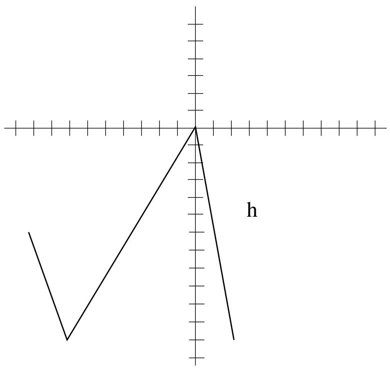
8. The graph of $-f(x)$ is a reflection of the graph of $f(x)$ over the x-axis.

9. (a) $3f$.
(b) f.
(c) $-3f$.

10.

x	-2	-1	0	1	2	3
$f(x)$	-5	-3	-1	2	4	8
$-3f(x)+1$	16	10	4	-5	-11	-23
$2f(x)-3$	-13	-9	-5	1	5	13

11.

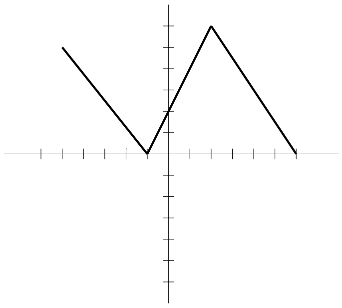


Solutions for Section 3.2 Reading Questions

1. A deduction reduces the amount of taxable income which is the input for the tax function.

2. $g(x) = f(x+1) = 2(x+1) - 7 = 2x - 5$. g is parallel to f but shifted one unit to the left.

3.



4. Compared to $y = f(x)$, the graph of $y = f(3x)$ has been horizontally compressed by a factor of three.

5.

x	-2	-1	0	1	2
$f(x)$	-6	-4	-2	0	2
$f(2x)$		-6	-2	2	

6. $t = 6$ hours

7. The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected over the y-axis.

8. (a) $y = f(-x)$.
(b) $y = f(-2x)$.
(c) $y = f(x)$.

9.

x	0	1	2	3	4
$f(x)$	2	4	8	11	13
$2f(x-1)+3$		7	11	19	25

10. $3(2)^{2x-1} + 7$

Solutions for Section 3.3 Reading Questions

1. $p(t) = 269,126,342 + \frac{1}{8}t - \frac{1}{18}t + \frac{1}{39}t + \frac{1}{4473}t \approx 269,126,342 + (0.0953)t$

2.

x	0	1	2	3	4
$u(x)$	6	9	13	18	24
$t(x)$	-3	-2	-1	0	1
$r(x)$	9	11	14	18	23

3. $(f+g)(3) = f(3) + g(3) = 3^2 + 2^3 = 9 + 8 = 17$

4. To add two functions you add their outputs. A change in output shifts the graph vertically.

5. It is difficult to observe the effect of $\sin x$ because Figure 4 consists of a graph with a large viewing window and the values of $\sin x$ are much smaller than $0.1x^2$ for large values of x.

6. $r(30,000) = \frac{0.17(30,000-13,300)}{30,000} = 0.0946 = 9.46\%$

7.

x	0	1	2	3	4
$s(x)$	6	9	13	18	24
$t(x)$	-3	-2	-1	0	1
$(s/t)(x)$	-2	$-\frac{9}{2}$	-13		24

8. Yes, f is undefined at $x = 5$.

9. $(f \cdot g)(3) = f(3) \cdot g(3) = 3^2 \cdot 2^3 = 72$

10. At points where x is negative, the graph of $y = x \sin x$ will be on the opposite side of the x-axis from $y = \sin x$ because multiplying by a negative is a reflection over the x-axis.

11. (a) $x \geq 0$.
(b) $x \geq 0$, $x \neq 2$.

12. The domain remains the same since the domain of a constant is all real numbers.

13. The domain of a function may include numbers which cause the value of the function to be 0. If that function is in the denominator of a quotient, such numbers cause the quotient to be undefined.

Solutions for Section 3.4 Reading Questions

1. It is a composition because the first function turns the bread ingredients into bread dough and the second function turns the bread dough into a loaf of bread. These two functions are linked together so that the output of the first function is the input of the second.

$$\text{ingredients} \xrightarrow{\text{bread machine}} \text{bread dough} \xrightarrow{\text{bread machine}} \text{loaf}$$

2. If $g(f(x))$ is defined, the range of f is a subset of the domain of g, however $f(g(x))$ will only be defined if the range of g is a subset of the domain of f. The latter will not necessarily be true.

3. $f(1) = \frac{1-3}{1} = -2 \rightarrow g(f(1)) = 2(-2)^3 = -16$.

4. (a) $g(0) = 1 \rightarrow h(g(0)) = h(1) = 0$.

(b) $h(-2) = 4 \rightarrow f(h(-2)) = f(4) = 2$.

(c) $f(9) = 3 \rightarrow g(f(9)) = g(3) = 10$.

(d) $h(-1) = 1 \rightarrow f(h(-1)) = f(1) = 1 \rightarrow g(f(h(-1))) = g(1) = 4$.

5. He will use $S(m) = \frac{m}{20.75}$ as the first function because dividing the combined length of the walls by by the width of each strip of wallpaper will give you the number of strips.

6. From Table 3, we see that $F(10pm) = 61$. Using the formula, $C(F) = \frac{5}{9}(F - 32) \rightarrow C(61) = \frac{5}{9}(61 - 32) \approx 16°C$.

7. Answers will vary. One example of a physical situation involving composition can be seen at a manufacturing plant where the input is raw plastic and the output is a small part which then becomes the input for a bigger piece of equipment.

8. The domain of $h(x) = g(f(x))$ is a subset of the domain of f (all real numbers except $x = -2$) whose outputs are valid inputs for g. The domain of g is all reals such that $x \neq 0$. $f(x) = 0$ when $x = 0$. Therefore, the domain of $h(x) = g(f(x))$ is all reals such that $x \neq 0$, $x \neq -2$.

9. The domain of $h(x) = g(f(x))$ must be a subset of the domain of f $(x \geq 0)$ whose outputs are valid inputs for g. The domain of g is $x \geq 0$ and all outputs of f are part of this domain so the domain of $h(x) = g(f(x))$ is $x \geq 0$. The range of g is $x \geq 0$ when the domain is the outputs of f so the range of $h(x) = g(f(x))$ is $x \geq 0$.

10. The domain of $k(x) = f(g(x))$ is a subset of the domain of g, $0, 1, 2, 3, 4, 5, 6$, whose outputs are valid inputs for f. The domain of f is $-3, -2, -1, 0, 1, 2, 3$ and the output of g is part of this domain except when $x = 1$. Therefore the domain of $k(x) = f(g(x))$ is $0, 2, 3, 4, 5, 6$. Over this domain, the possible outputs of g, or possible inputs of f are $-1, 0, 1, 2, 3$. These inputs for f produce outputs of $-2, -1, 0, 1, 2$ so this set is the range of $k(x)f(g(x))$.

11. One solution is $f(x) = x^2 + 1$ and $g(x) = 10^x$.

12. One solution is $f(x) = x^2$ and $g(x) = 3 - \frac{1}{x}$. Another solution is $f(x) = -\frac{1}{x^2}$ and $g(x) = 3 + x$.

Solutions for Section 3.5 Reading Questions

1. The inverse would be subtracting 12 from the input and then dividing by 5.

2. The input of the function is the output of its inverse and the output of the function is the input of the inverse.

3. Answers will vary. One example of an appropriate answer is, "An inverse function is a function which reverses the effect of an original function."

4. It denotes that the function is an inverse.

5. Solving for x gives us $\frac{x}{3} = f(x) + \frac{4}{3} \rightarrow x = 3f(x) + 4$. Now rewriting with $f^{-1}(x)$ in place of x and x in place of $f(x)$, we get $f^{-1}(x) = 3x + 4$

6.

x	-2	0	1	3	4
$g^{-1}(x)$	2	1	0	-1	-2

7. A function and its inverse are reflections of each other over the line $y = x$.

8. Some functions do not have inverses because they have more than one input which produces the same output. The inverse of this is not a function.

9. A one-to-one function is a function in which each output has only one input.

10. If any horizontal line crosses the graph of a function more than once, the function is not one-to-one (there is more than one input for some outputs) and does not have an inverse.

11. The domain of a function that is not one-to-one can be restricted so that each output has exactly one input.

12. The functions are both periodic but are horizontal shifts of each other. If we used the restricted domain of one function on the other, it would not pass the horizontal line test.

13. $-3 < x < -1.5$ or $-1.5 < x < 0$ or $0 < x < 3.8$ or $3.8 < x < 5$