Solutions for Section 2.1 Reading Questions

1. A linear function is a function in which the rate of change (which is equal to $\frac{\text{change in output}}{\text{change in input}}$) is constant.

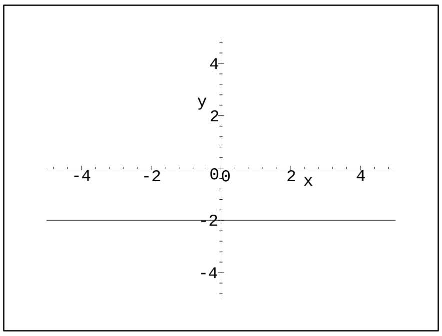
2. The two ways of thinking about the y-intercept are actually the same because when the input is zero, the output lies on the y-axis.

3. The slope of the line is $\frac{6-3}{2-1} = 3$.

4. The slope of a linear function that decreases is negative.

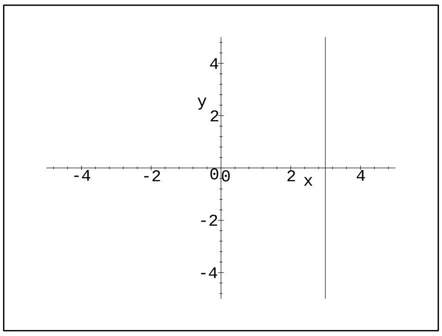
5. The y-intercept of the linear function is $f(0) = 3(0) + 4 = 4$.

6. (a)



$y = -2$

(b)



$x = 4$

7. If you are given a graph, you can determine its equation by estimating the constant slope and the point at which the graph crosses the y-axis.

8. The slope of the function appears to be $\frac{0-(-4)}{2-0} = 2$, and y-intercept is -4. Therefore, the equation of the linear function is $y = 2x - 4$.

9. By using the point $(4, 0)$ instead of (2,3), we would get the same equation $y = -\frac{3}{2}x + 6$, because the equation of a line has to work for all the points on your line.

10. We determine if a function is linear from a given table of data, by determining if the function has a constant slope. We do this by making sure that the $\frac{\text{change in output}}{\text{change in input}}$ is constant.

11. (a) It is a linear function. The equation of the linear function is $y = 4x + 18$.

(b) It is a linear function. The equation of the linear function is $y = 2x + 1$.

12. Verbally, we determine if a function is linear if a situation has a constant rate of change.

13. (a) (Total Wages)= $6.25 \cdot$ hours.

(b) (number of yards)= $\frac{1}{36} \cdot$ (number of inches).

14. (a) In the equation which converts from degrees Fahrenheit to degrees Celsius ($C = \frac{5}{9}F - 17\frac{7}{9}$), $\frac{5}{9}$ is the rate of change or slope.

(b) In the equation which converts from degrees Fahrenheit to degrees Celsius ($C = \frac{5}{9}F - 17\frac{7}{9}$), $-17\frac{7}{9}$ is the point in Celsius degrees at which Fahrenheit is zero.

15. In both cases, you are making judgements about the entire function based on just some of the points.

16. To determine if a symbolic representation is a linear function, you should solve for y, and write the function in a slope-intercept form.

17. (a) It is a linear function. The equation is $y = 3x + 1$.

(b) It is a linear function. The equation is $y = \frac{1}{3}x + \frac{10}{3}$.

(c) It is a linear function. The equation is $y = \frac{1}{2}x - 3$.

Solutions for Section 2.2 Reading Questions

1. After participating in 10 regular seasons and 12 playoff games, the baseball player received \$8,388,608 for playing the first game of the World Series.

2. Exponential functions are similar to linear functions in that they both have a constant rate. However, a linear function has a constant rate of change while an exponential function has a constant growth factor.

3. $f(0) = 3 \cdot 4^0 \rightarrow$ y-intercept $= 3$.
Growth factor= $\frac{f(x+1)}{f(x)} \rightarrow \frac{f(2)}{f(1)} = \frac{48}{12} = 4$.

4. Every exponential function passes through $(0, 1)$ because, $f(0) = a^0 = 1$.

5. For an exponential function of the form $E(x) = a^x$, it is necessary that a be positive. If a were negative, there would be some values of x for which $E(x) = a^x$ would be undefined.

6. If $f(x) = a^x$ and $f(1) = 6$ then $a^1 = 6$ and $a = 6$.

7. One is excluded as a growth factor because 1 raised to any power is still 1 ($f(x) = 1^x = 1$). With 1 as a growth factor, the exponential function would not *grow* exponentially but would remain at 1.

8. $a < 1$ because the function is decreasing.

9. b is the y-intercept because $f(0) = ba^0 = b$. Therefore, when x is zero, the exponential function intersects the y-axis at b.

10. $f(x) = 2 \cdot 5^x$.

11. $b = 3$ because it is the y-intercept.

12. $f(n) = 500 \cdot 2^n$, where n is the number of games played.

13. (a) It is an exponential function. $y = \frac{1}{3} \cdot (\frac{1}{4})^x$.

(b) It is not an exponential function because the growth factor is not constant.

(c) It appears to be an exponential function. The y-intercept is 2 (so $b = 2$) and it appears as though it contains the point $(1, \frac{2}{3})$. Plugging in the point $(1, \frac{2}{3})$ and $b = 2$ into $y = b \cdot a^x$, we get $\frac{2}{3} = 2 \cdot a^1$. Solving this for a, we get $a = \frac{1}{3}$. So the equation is $y = 2 \cdot (\frac{1}{3})^x$.

(d) It is not an exponential function, but a linear function $y = 0.10x$.

14. Bacteria doubling every 6 hours is equivalent to 16 times a day because, if it doubles every 6 hours, there are 4 times as many after 12 hours, 8 times as many after 18 hours, and 16 times as many after 24 hours (which is equivalent to 1 day).

15. Tuition usually increases only at the beginning of a new school year. So after $3\frac{1}{2}$ years, the tuition would probably cost the same as at 3 years.

Solutions for Section 2.3 Reading Questions

1. Magnitude of a number a is the exponent b such that $10^b = a$.

2. $10^b = 1,000,000 \to b = 6$, where b is the magnitude of $1,000,000$.

3. $10^b = 6,132,678 \to b = 6.78$, where b is the magnitude of $6,132,678$.

4. The range of the graph of the logarithm function for $1 \leq x \leq 1000$ is $0 \leq y \leq 3$.
The graph of $y = \log x$ increases gradually over the domain $100 \leq x \leq 1000$.

5. The logarithm function is concave down because the slope (rate of change) decreases as x increases.

6. If $\log r$ is negative, $0 < r < 1$.

7. It is impossible to find a number p, such that $10^p = -4$, because 10 raised to any power cannot give a negative output.

8. $\log 140 \approx 2.146$ since an increase of 10 in the input of the log function causes an increase of 1 in the output.

9. Since $16 = 4^2$, $\log 16 = \log 4^2 = 2 \log 4 \approx 2 \times 0.602 = 1.204$.

10. Since $2 = 4^{1/2}$, $\log 2 = \log 4^{1/2} = \frac{1}{2} \log 4 \approx \frac{1}{2} \times 0.602 = 0.301$.

11. Measuring in magnitudes is useful when trying to perceive the difference in large quantities, because it is easy to distinguish between magnitudes of change, while it is often difficult to distinguish change measured in differences.

12. The growth factor for the exponential function whose input is the number of decibels and whose output is the sound level is 1.26.

13. Both the logarithm functions and exponential functions deal with exponents. For exponential functions, the exponent is the input, and for logarithmic functions, the exponent is the output.

14. Answers will vary. A possible physical situation for which a logarithm function might be a good model is the growth of a tree. A young tree will grow quickly and as it gets older, it will slow down.

Solutions for Section 2.4 Reading Questions

1. A periodic function is one that gives the same output for inputs a fixed distance apart.

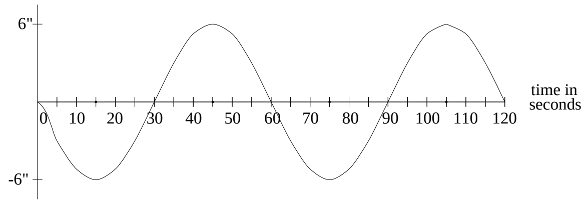
2. Graph (d) in Figure 1 is not a function because it gives more than one output for a single input.

3. Answers will vary.

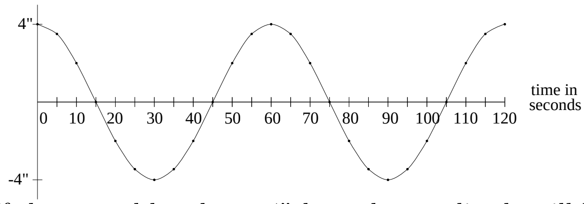
4. $f(5) = -2$.

5. The period of the function describing the vertical position of the second hand of a clock is 60 seconds.

6.

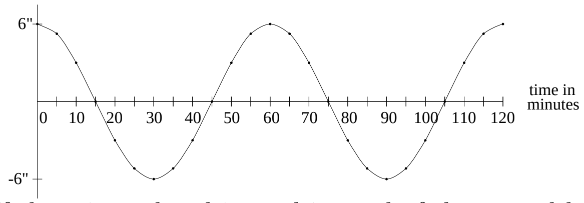


7.



If the second hand was 4" long the amplitude will be reduced from 6" to 4".

8.



If the minute hand is used instead of the second hand, the period will change from 60 seconds to 60 minutes.

9. The vertical position function and the horizontal position function of the swing have different periods because, for the vertical position function, we look at the swing's minimum and maximum height. The swing goes from the highest point to the lowest point and back to the highest again. This makes the vertical position function's period 1 second. For the horizontal position function, we look at when the swing is farthest to the right and farthest to the left. The swing goes from A (farthest to the right) to B (farthest to the left) and back to A. This makes the horizontal position function's period 2 seconds.

10. If, for instance, someone pushes Emma from point B, which makes her swing go beyond point A, it will change the amplitude of the horizontal function.

11. The only way you could change the period would be to change the length of the swing.

12. The circumference of a circle is $2\pi r$. If $r = 1$ then the circumference, or one revolution around the circle, is $2\pi \cdot 1 = 2\pi$.

13. $\sin x$ is the vertical position of a point that is x units in the counterclockwise direction from the point $(1, 0)$ on the unit circle.

14. $\cos x$ is the horizontal position of a point that is x units in the counterclockwise direction from the point $(1, 0)$ on the unit circle.

15. In order to show that the right triangle definition gives the same function as the unit circle definition of $\sin x$, it is important to use a unit circle because then the measure of the central angle is the same as the intercepted arc.

16. The radian measure of an angle is the length of the arc on the unit circle intercepted by that angle.

17. Radian measures are important because, the right triangle definitions and the unit circle definitions give the same answer when radian measures are being used for the input.

18. $\frac{\pi}{2}$

Solutions for Section 2.5 Reading Questions

1. Something that can be modeled with a linear function has a constant rate of change while something that can be modeled with a power function does not have a constant rate of change.

2. If a function is given symbolically in the form of $f(x) = bx^a$, then it is a power function.

3. (a) $y = x^{15}$. It is an odd power because it gives negative outputs for negative inputs. It also increases faster than (c) for inputs greater than 1, so it will be the larger odd power.

(b) $y = x^4$. It is an even power because it gives positive outputs for negative inputs. It also increases slower than (d) for inputs greater than 1, so it will be the smaller even power.

(c) $y = x^5$. It is an odd power because it gives negative outputs for negative inputs. It also increases slower than (a) for inputs greater than 1, so it will be the smaller odd power.

(d) $y = x^{10}$. It is an even power because it gives positive outputs for negative inputs. It also increases faster than (b) for inputs greater than 1, so it will be the larger even power.

4. In $f(x) = x^a$, where a is a positive integer, $f(1) = 1$ because 1 multiplied by itself will always be 1.

5. If a is odd, $f(-1)$ will equal -1, but if a is even, $f(-1)$ will equal 1.

6. At $(0, 0)$, the function switches from being concave down to concave up or vice versa.

7. The function does not change concavity at any point.

8. (a) $y = x^{1/4}$.

(b) $y = x^{4/5}$.

(c) $y = x^{9/5}$.

(d) $y = x^{6/5}$.

(e) $y = x^{3/5}$.

9. In $f(x) = x^a$, where a is a positive rational number, $f(0) = 0$ because zero raised to a positive rational number will always be zero. Similarly, $f(1) = 1$ because one raised to a positive rational number will always be one.

10. (a) For $y = x^{3/4}$, the domain is $x \geq 0$, and the range is $y \geq 0$.

(b) For $y = x^{9/8}$, the domain is $x \geq 0$, and the range is $y \geq 0$.

(c) For $y = x^{2/7}$, the domain is all real numbers, and the range is $y \geq 0$.

(d) For $y = x^{13/7}$, the domain is all real numbers, and the range is all real numbers.

11. If a graph of a power function is concave up in the first quadrant, the exponent is greater than 1.

12. If a graph of a power function is concave down in the first quadrant, the exponent is less than 1.

13. b makes the graph of $y = bx^a$ steeper if b is > 1, or flatter if b is < 1.

14. (a) When $x = 1$, $y = 5$, therefore, $b = 5$.

(b) When $x = 1$, $y = 2$, therefore, $b = 2$.

(c) When $x = 1$, $y = 3$, therefore, $b = 3$.

15. (a) $b = 4$.

(b) $32 = 4 \cdot 2^a \rightarrow 8 = 2^a \rightarrow a = 3$.

(c) $4.76 = 4 \cdot 2^a \rightarrow 1.19 = 2^a \rightarrow a \approx \frac{1}{4}$.

16. Kepler's third law is a power function because its rate of change is not constant and it is in the form of $f(d) = bd^a$, where $b = K$ and $a = \frac{3}{2}$.

17. $t = 1 \cdot 30.06^{3/2} \rightarrow t = 164.81$. It is approximately equal to the value in Table 9.