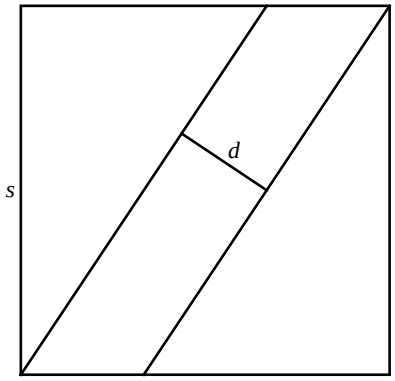
1. In the picture below, the two parallel cuts divide the square into three pieces of equal area. The distance between the two parallel cuts is d. The square has length s. Find and prove a formula that expresses s as a function of d.



2. Let S be a subset of $\{1, 2, 3, 4, \ldots, 10, 11\}$. We say that S is *lucky* if no two elements of S differ by 4 or 7.

   (a) Give an example of a lucky set with five elements.

   (b) Is it possible to find a lucky set with six elements? Explain why or why not.

3. Find polynomials $p(x)$ and $q(x)$ with real coefficients such that

   (a) $p(x) - q(x) = x^3 + x^2 - x - 1$ for all real x;

   (b) $p(x) > 0$ for all real x;

   (c) $q(x) > 0$ for all real x.

4. A permutation on $\{1, 2, 3, \ldots, n\}$ is a rearrangement of the symbols. For example 32154 is a permutation on $\{1, 2, 3, 4, 5\}$. Given a permutation $a_1a_2a_3 \ldots a_n$, an *inversion* is a pair of a_i and a_j such that $a_i > a_j$ but $i < j$. For example, 32154 has 4 inversions. Suppose you are only allowed to exchange adjacent symbols. For any permutation, show that the minimum number of exchanges required to put all the symbols in their natural positions (that is, $123 \ldots n$) is the number of inversions.

5. Say that the number N is a nontrivial sum of consecutive positive integers if N can be expressed as a sum of 2 or more consecutive positive integers. Determine, with proof, the set of all integers N between 1000 and 2000 which are **not** nontrivial sums of consecutive integers.

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