THIRTY-SEVENTH ANNUAL MICHIGAN MATH PRIZE COMPETITION

SOLUTION KEY

The answer key to Part One of the Thirty-Seventh annual Michigan Math Prize Competition, which was given on October 13, 1993 is as follows:

1. B	11. C	21. C	31. B
2. D	12. C	22. C	32. D
3. C	13. A	23. B	33. B
4. C	14. A	24. D	34. C
5. D	15. B	25. D	35. E
6. D	16. E	26. B	36. D
7. E	17. B	27. E	37. A
8. E	18. D	28. A	38. A
9. E	19. A	29. E	39. B
10. E	20. C	30. A	40. C

4. If x = the amount the price is decreased, and y the original number of tickets, then $(6 - x)(1.25y) = (1.125)(6y)$. If you like fractions, convert 1.25 to 5/4 and 1.125 to 9/8.

13. Observe that $5 + 2\sqrt{6}$ is $[\sqrt{3} + \sqrt{2}]^2$ --or write $\sqrt{5 + \sqrt{24}} = \sqrt{5 + 2\sqrt{6}} = \sqrt{3 + 2\sqrt{3}\sqrt{2} + 2}$.

14. See diagram below.

15. $a_0 = a_1 = 1$, $a_2 = a_0 + a_1 = 2$, $a_3 = 1 + 1 + 2 = 4 = 2^2$, $a_4 = 1 + 1 + 2 + 2^2 = 2^3$, ... $a_n = 2^{n-1}$.

17. In each 24 hours, there are 11 occurrences of coincidence between the hands, from 1:05 (the first) to 12:00 (the last). But to end before midnight Monday, you need to eliminate one of them to get 21 in all.

18. |<-4-><--------10 -------->| The small circle has radius 4, giving a ratio of areas $16\pi / 100\pi$.
<---------14 ------------->

20. Let $z = \log_{10} x$ and $w = \log_{10} y$. Then a line in the zw- plane, $w = mz + b$, in the xy-plane becomes $\log_{10} y = m(\log_{10} x) + b$. Write $b = \log_{10} 10^b$ and use rules of logs to give $y = cx^m$, $c = 10^b$

21. $1/x + 1/y = (x + y) / xy = 8/14 = 4/7$. Simultaneous solution of $x + y = 8$ and $xy = 14$ also works.

22. (1)(3)(5)(7)(9) = 945 so the last digit will be a 5.

24. Let E be the intersection of AC and BD. Then BE = 6 (a nice use of the 3-4-5 right triangle).

25. Look at the answers <u>carefully</u>. Then check the best bet (#d !)

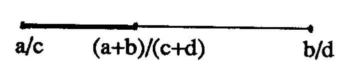
27. The line $x = m$ divides the x-axis into two pieces, of lengths m and 1-m, It divides the triangle into a trapezoid with area $A_1 = (m/2)[1 + (1 - m)]$ and a triangle with area $A_2 = (1/2)(1 - m)(1 - m)$.

28. See diagram below.

30. Let r = radius of inner circle. Use symmetry to draw a 30-60-90 right triangle (see diagram below). The leg opposite the 60° angle is the radius of the large circle, length 1. So the hypotenuse is length $1 + r = 2/\sqrt{3}$.

31. Write $2 = 7^{\log_7 2}$. Simplification of exponents gives $7^{\log_7 3} = 3$.

32. $\left(\frac{a+b}{c+d} - \frac{a}{c}\right) \Big/ \left(\frac{b}{d} - \frac{a}{c}\right)$ Draw a number line to visualize the differences.



33. A nice counting problem. There are 1(9) + 2(90) + 3(100) + 3(100) + 3(70) = 789 digits in the numbers up to 999. The first digit in the next number (370) is a 3.

34. $162 = 2 \times 3^4$. There are 50 multiples of 2 and 33 multiples of 3 in the numbers from 1 to 100. So 83/100 are irreducible.

35. See diagram below.

36. Insert the missing terms and subtract them to generate 2 geometric series: $(1/2)\sum_0^\infty(1/2)^n - (1/2^3)\sum_0^\infty(1/2^3)^n$.

37. The water volume is less than 1/4 the total tank volume (see diagram at right). So when turned upright, the height x of the water is less than 1/4 of the total height of 8 or $x < 2$ ft. (A calculation shows $x = 8/3 - \sqrt{3}/\pi$.)

38. $(1/1000)\left[\sum_1^N n\right] < 1$ gives $[N(N+1)/2] < 1000$ or $N(N+1) < 2000$. A good guess for N would be between 40 and 50.

39. A quick elimination gives the number 59.

40. Let R:= Right triangle, I:= Isosceles triangle, >2:= Side greater that 2 in.

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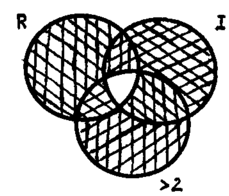
betish hatching ///
gammish hatching |||

The Venn diagram for the sets will look like this:



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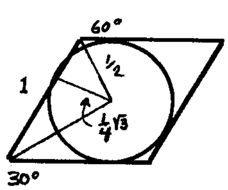

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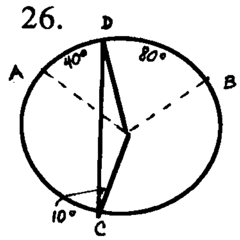

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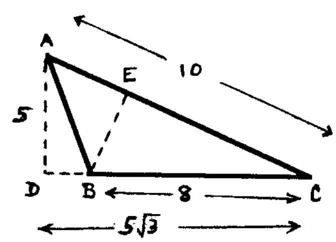

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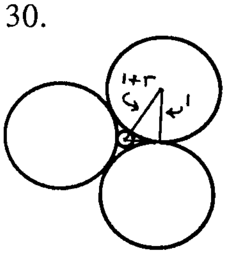

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