

# THIRTY-SIXTH ANNUAL MICHIGAN MATH PRIZE COMPETITION

## SOLUTION KEY

The answer key to Part One of the Thirty-Sixth annual Michigan Math Prize Competition, which was given on October 14, 1992 is as follows:

1. E	11. A	21. E	31. B
2. D	12. E	22. B	32. C
3. E	13. D	23. C	33. D
4. B	14. C	24. C	34. B
5. D	15. A	25. A	35. D
6. D	16. B	26. D	36. C
7. D	17. C	27. D	37. B
8. D	18. A	28. B	38. C
9. D	19. A	29. A	39. B
10. E	20. B	30. A	40. C

6. Each term  $s_k$  in the series is such that  $.0001 \leq s_k < .001$ . There are 9000 terms, so  $.9 \leq S < 9$ .

11. The probability that the top half has as many black as the bottom half has red cards is 1, so that the probability that the top has more is 0.

16. Observe that  $A_2M_3M_2M_1$  is a parallelogram, and that  $M_3M_2 \parallel A_2A_3$ , and  $M_1M_2 \parallel A_2A_1$ . Area  $A_1M_3M_2$  is  $1/9$  of the total area (see problem #3!) and area  $A_1A_2M_1M_2$  is  $5/9$  of total area, so area  $M_1M_2M_3$  is  $1/2(4/9) = 2/9$  of the total area.

21. Four choices for the first toy, 3 for the second, 2 for the third, and 3 ways to give the fourth toy. Divide by 2 to eliminate duplicates:  $[(4 \times 3 \times 2) \times 3] / 2 = 36$ .

22. See diagram below.

25. If  $ac + bd > ad + bd$  then  $(ac + bd) - (ad + bd) > 0$ . Factor:  $(a - b)(c - d) > 0$ . This is always true, since we are given  $a > b$  and  $c > d$ .

26. Guess and check is one way, starting with the sequence 2, 3, 7 (which doesn't work) and discovering 20, 21, 25 (and hence others) in short order. Alternatively, one can use modular arithmetic.

27. See diagram below for the case of a pentagonal cross section, which may not be obvious.

28. The three rational answers fit one of the possibilities for a root, as does the product of the irrational answers. That latter product is positive, so the product of the other two roots must also be positive, eliminating 1.5.

31. See diagram below. One easy way to do this is to observe that  $X$  is the area of a semicircle plus area  $(1/2)Y$ .

32. An unstated assumption is that the earth is a sphere. All lines cut off a chord of some great circle of a sphere, and hence are perpendicular to a diameter, so all lines are horizontal by our definition and I is true. All diameters are perpendicular to some other diameter, so all of our vertical lines are horizontal and II is true. But there are some lines which don't pass through the center, so III is false.

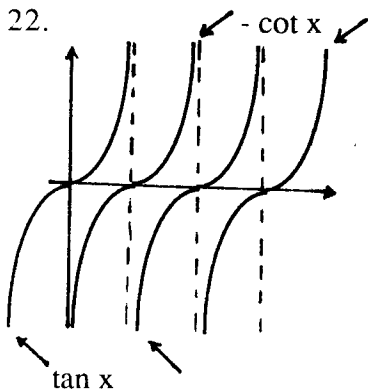
35. The 71 numbers from 0 to 70 are one set which fits the description, and 3 of them are divisible by 35.

37.  $P(\text{both red}) = (2/9)(1/8)$ ;  $P(\text{both blue}) = (3/9)(2/8)$ ;  $P(\text{both green}) = (4/9)(3/8)$ . Add these probabilities.

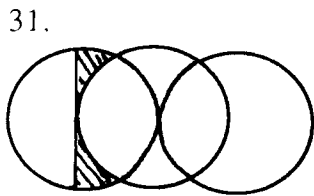
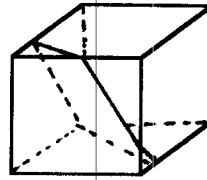
38. See diagram below.

39.  $((7^7)^x)^{1/7} = ((7^x)^7)^{1/7} = 7^x$

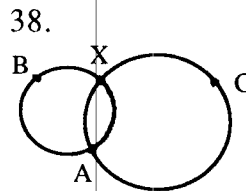
40. Aren't you glad this one was last? Guess and check is a quick way out of perplexity (one set that works is 5, 10, 20), but not the only way. Let A, B, C be the ages. We know  $C = 2B$  and  $B + A = C - A$ . So  $2A = B$  and the ratio is 2:1.



27. A pentagonal cross-section is shown.



The shaded portion represents  $(1/2)Y$



Angle AXC is a right angle since AC is a diameter.  
 Angle AXB is a right angle since AB is a diameter.  
 Therefore BXC is a straight line.