

October 25, 1989

Dear MMPC Supervisor:

The answer key to Part One of the Thirty-Third annual Michigan Math Prize Competition, which was given on October 11, 1989 is as follows:

1. B	11. E	21. C	31. D
2. E	12. A	22. C	32. B
3. C	13. D	23. D	33. A
4. C	14. B	24. A	34. D
5. B	15. A	25. E	35. E
6. B	16. C	26. C	36. E
7. A	17. C	27. C	37. E
8. B	18. B	28. D	38. A
9. E	19. D	29. D	39. B
10. D	20. D	30. A	40. C

A few comments regarding some of the questions follow:

- The graphs of $y = \cos^2\theta$ and $y = \sin^2\theta$ must cross somewhere between $\theta = 0^\circ$ and $\theta = 90^\circ$.
- Since it is a right triangle inscribed in a circle, the hypotenuse is a diameter.
- Consider the situation after he has drawn out one sock.
- Each of a, b and c must be 1 or -1.
- The arc is a quarter the circumference of the circle, and so its central angle is 90° .
- The polynomial factors as $(x^2-1)(x^2-2)(x^2+1)$.
- Separating x and y to opposite sides, adding 1 to both sides and factoring results in $(x-1)^2 = (y-1)^2$. So $x-1 = y-1$ or $x-1 = -(y-1)$.
- The thing to note here is that if you begin by multiplying both sides by x, the inequality reverses if $x < 0$.
- The total number of different choices is $4 \cdot 4 \cdot 4$, while the number for which the three hotels are different is $4 \cdot 3 \cdot 2$.
- The hypotenuse of the triangle is $2h$, so its area is h^2 , which leads to $s^2 = 4h^2$.
- The car travels 360 kilometers in 5 hours.
- The graph of $|y| = x + 1$ is two straight lines, and the circle given by the other equation goes through their point of intersection.
- x satisfies the equation $x = \sqrt{1+x}$; square both sides and solve; only one solution is positive.
- Words fail me -- see over.
- Join A to each vertex of the triangle. If the side of the triangle is s, the three triangles you have just created have area $sx/2$, $sy/2$ and $sz/2$. Since the three areas add to the total area of the equilateral triangle, the result follows. (Note that the sum $x + y + z$ does not depend on the location of the point A; if you are prepared to accept this without the above construction, consider then what happens as you move A towards a vertex of the triangle.)
- You can see this by graphing $y = \cos x$ and $y = \sin 2x$ from 0° to 360° .
- If n is even, $x^{mn}-1$ has factors x^m-1 and x^{m+1} , while if n is odd $x^{mn}-1$ has factor x^m-1 but not x^{m+1} . The question implies that one of the five choices does not divide $2^{10}-1$, so it must be e) by elimination of the other four choices (indeed, 1025 does not divide 1073741823).
- This is my favorite. Note that (1,0) and (0,1) must be on the graph. This gives an idea of the scales. Since $1-x^{1989}$ has a 1989th-root for any x, a), b) and e) are ruled out. Now if we let $x = y$ we see that $x^{1989} = 1/2$, so x must be very close to 1; i.e., the graph goes very close to (1,1). [When $x = y$, x is approx. .99965]. On the other hand, when x is large then y is very close to -x, since x^{1989} and y^{1989} are very large numbers whose sum is 1.

I am currently processing the (roughly 20,000) answer sheets (I've already found one 40). Thank you for your help and cooperation.

Chris Gardiner

