

**THIRTY-SECOND ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION**

sponsored by
The Michigan Section of the Mathematical Association of America

Part II

December 12, 1988

INSTRUCTIONS

(to be read aloud to the students by the supervisor or proctor)

1. Carefully record your six-digit MMPC code number in the upper right-hand corner of this page. This is the only way to identify you with this test booklet. **PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.**
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all the problems and work first on those that interest you the most.
4. Each problem is on a separate page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem number in the upper right-hand corner of each additional sheet.
5. If you are unable to completely solve a particular problem, partial credit may be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you given an explanation of why the generalization might be true.
7. The competition rules prohibit your asking questions of anyone during the examination. The use of notes, reference materials, computational aids, or any other aid is likewise prohibited. When the supervisor announces that the 100 minutes have elapsed, please cease work immediately and insert all significant extra paper into the test booklet. It is not necessary to return scratch paper on which routine numerical calculations have been made.
8. You may now open the test booklet and begin.

Score:

_____	_____	_____	_____	_____	_____	_____
1	2	3	4	5	TOTAL	

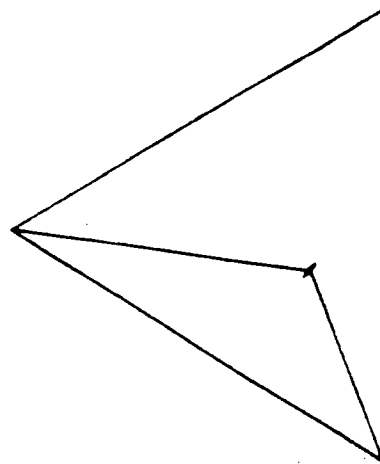
1.-

Given an equilateral triangle ABC with area $16\sqrt{3}$, and an interior point P with distances from vertices $|AP| = 4$ and $|BP| = 6$.

[3 points] (a) Find the length of each side.

[3 points] (b) Find the distance from point P to the side AB .

[4 points] (c) Find the distance $|PC|$.



2.-

Several players play the following game. They form a circle and each in turn tosses a fair coin. If the coin comes up heads, that player drops out of the game and the circle becomes smaller, if it comes up tails that player remains in the game until his or her next turn to toss. When only one player is left, he or she is the winner. For convenience let us name them A (who tosses first), B (second), C (third, if there is a third), etc.

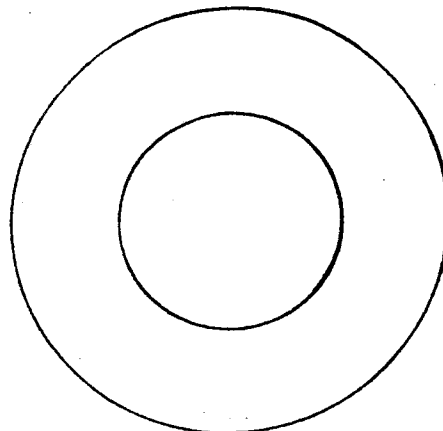
- [4 points] (a) If there are only two players, what is the probability that A (the first) wins?
- [4 points] (b) If there are exactly 3 players, what is the probability that A (the first) wins?
- [2 points] (c) If there are exactly 3 players, what is the probability that B (the second) wins?

3.-

A circular castle of radius r is surrounded by a circular moat of width m (m is the shortest distance from each point of the castle wall to its nearest point on shore outside the moat). Life guards are to be placed around the outer edge of the moat, so that at least one life guard can see anyone swimming in the moat.

[3 points] (a) If the radius r is 140 feet and there are only 3 life guards available, what is the minimum possible width of moat they can watch?

[7 points] (b) Find the minimum number of life guards needed as a function of r and m .



4.-

- [2 points] (a) Find all linear (first degree or less) polynomials $f(x)$ with the property that $f(g(x)) = g(f(x))$ for all linear polynomials $g(x)$.
- [2 points] (b) Prove your answer to part (a).
- [2 points] (c) Find all polynomials $f(x)$ with the property that $f(g(x)) = g(f(x))$ for all polynomials $g(x)$.
- [4 points] (d) Prove your answer to part (c).

5.-

A non-empty set B of integers has the following two properties:

- i. each number x in the set can be written as a sum $x = y + z$ for some y and z in the set B .
(Warning: y and z may or may not be distinct for a given x .)
- ii. the number 0 can not be written as a sum $0 = y + z$ for any y and z in the set B .

[2 points] (a) Find such a set B with exactly 6 elements.

[2 points] (b) Find such a set B with exactly 6 elements, and such that the sum of all the 6 elements is 1988.

[2 points] (c) What is the smallest possible size of such a set B ?

[4 points] (d) Prove your answer to part (c).

The Michigan Mathematics Prize Competition
is an activity of the
Michigan Section of the Mathematical Association of America

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The Michigan Association of Secondary School Principals has placed this competition on the Approved List of Michigan Contests and Activities.