MMPC CODE NUMBER

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TWENTY-NINTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MICHIGAN SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA

PART II

December 11, 1985

INSTRUCTIONS
(to be read aloud to participants by supervisor or proctor)

1. Carefully record your six digit MMPC code number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.

3. You are not expected to solve all problems completely. Look over all the problems and work first on those that interest you the most.

4. Each problem is on a separate page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem number in the upper righthand corner of each additional sheet.

5. If you are unable to completely solve a particular problem, partial credit may be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. The competition rules prohibit your asking questions of anyone during the examination. The use of notes, reference material, computational aids, or any other aid is likewise prohibited. When the supervisor announces that the 100 minutes have elapsed, please cease work immediately and insert all significant extra paper into the booklet. It is not necessary to return scratch paper on which routine numerical calculations have been made.

8. You may now open the test booklet and begin.

Score	___	___	___	___	___	___
	1	2	3	4	5	TOTAL

1. Sometimes one finds in an old park a tetrahedral pile of cannon balls, that is, a pile each layer of which is a tightly packed triangular layer of balls.

(3 points) A. How many cannon balls are in a tetrahedral pile of cannon balls of N layers?

(7 points) B. How high is a tetrahedral pile of cannon balls of N layers? (Assume each cannon ball is a sphere of radius R.)

2\. A prime is an integer greater than 1 whose only positive integer divisors are itself and 1.

(3 points) A. Find a triple of primes (p, q, r) such that $p = q + 2$ and $q = r + 2$.

(7 points) B. Prove that there is only one triple (p, q, r) of primes such that $p = q + 2$ and $q = r + 2$.

3. The function g is defined recursively on the positive integers by $g(1) = 1$, and for $n > 1$, $g(n) = 1 + g(n - g(n - 1))$.

(2 points) A. Find $g(1)$, $g(2)$, $g(3)$ and $g(4)$.

(3 points) B. Describe the pattern formed by the entire sequence $g(1)$, $g(2)$, $g(3)$, . . .

(5 points) C. Prove your answer to Part B.

4. Let x, y, and z be <u>real</u> numbers such that

$$x + y + z = 1 \quad \text{and} \quad xyz = 3 .$$

(2 points) A. Prove that none of x, y, nor z can equal 1.

(8 points) B. Determine all values of x that can occur in a simultaneous solution to these two equations (where x, y, z are real numbers).

5. A round robin tournament was played among thirteen teams. Each team played every other team exactly once. At the conclusion of the tournament, it happened that each team had won six games and lost six games.

(2 points) A. How many games were played in this tournament?

(3 points) B. Define a circular triangle in a round robin tournament to be a set of three different teams in which none of the three teams beat both of the other two teams. How many circular triangles are there in this tournament?

(5 points) C. Prove your answer to Part B.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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