

28th Annual Michigan Mathematics Prize Competition

Part I Answers

| <u>Key</u> | <u>Comments</u> |
|------------|---|
| 1. C | |
| 2. A | |
| 3. C | |
| 4. D | $x = -1$ is an element of this set |
| 5. D | eg in base 2, $8 = 1000$ and $16 = 10,000$ |
| 6. B | There are 10 palindromes each between 1000 and 2000, 2,000 and 3,000, etc. |
| 7. E | In fact, $x = \pm \frac{(1+i)}{\sqrt{2}}$ |
| 8. D | |
| 9. B | |
| 10. A | |
| 11. E | |
| 12. B | |
| 13. D | Note that $N_k = \begin{cases} 1, & k \text{ even} \\ -1, & k \text{ odd} \end{cases}$ |
| 14. D | |
| 15. B | |
| 16. E | If $x \neq 1$, then $1 + x + x^2 + \dots + x^{n-1} = \frac{x^n - 1}{x - 1}$ |
| 17. D | $4.\overline{326} = 4.3 + \frac{26}{10^3} + \frac{26}{10^5} + \dots$ and $a + ar + ar^2 + \dots = \frac{a}{1-r}$ if $ r < 1$ |
| 18. A | If any polynomial $P(x)$ is divided by $x-a$, then $P(a)$ is the remainder. |
| 19. D | $x = y$ if and only if $\log x = \log y$ ($x, y > 0$) |
| 20. E | |
| 21. E | $e^t > 0$ for all real t . |
| 22. E | Starting from overlapping position, the minute hand gains $5 \frac{1}{2}^\circ/\text{min.}$ over the hour hand. |
| 23. B | $\log_b a = c$ means $b^c = a$. |
| 24. D | |
| 25. D | |
| 26. B | |
| 27. C | |
| 28. D | |
| 29. C | |
| 30. E | $\cos 2\theta = 2\cos^2\theta - 1$ yields $\cos \theta = \frac{1 - \sqrt{3}}{2} < 0$. |

31. D

32. D

33. E

34. E

Eg. $A = \{2\}$, $B = \{2,3\}$

35. B

36. A

37. A

Angle at the center = twice the angle at circumference

38. E

$$4\theta = \frac{\pi}{4} + k\pi, k = 0,1,2,\dots,7$$

39. E

40. E