MMPC CODE NUMBER

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TWENTY-SIXTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 8, 1982

INSTRUCTIONS
(to be read aloud to class by supervisor or proctor)

1. Carefully record your six digit MMPC code number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.

3. You are not expected to solve all problems completely. Look over all the problems and work first on those which interest you the most.

4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem in the upper righthand corner of each additional sheet.

5. If you are unable to completely solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. The competition rules do not allow anyone to answer any questions. The use of notes, reference material, computational aids, or any other aid is prohibited. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

8. You may now open the test booklet and begin.

Score					
1	2	3	4	5	TOTAL

1. Sarah needed a ride home to the farm from town. She telephoned for her father to come and get her with the pickup truck. Being eager to get home, she began walking toward the farm as soon as she hung up the phone. However, her father had to finish milking the cows, so could not leave to get her until fifteen minutes after she called. He drove rapidly to make up for lost time.

They met on the road, turned right around and drove back to the farm at two-thirds of the speed her father drove coming. They got to the farm two hours after she had called. She walked and he drove both ways at constant rates of speed.

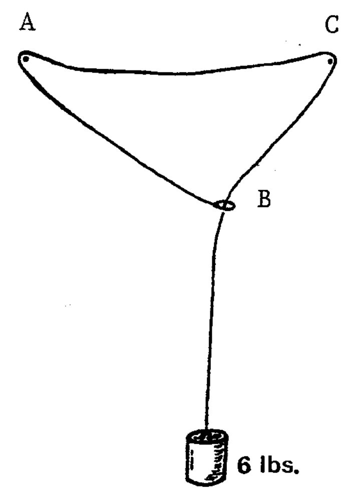
How many minutes did she spend walking?

2. Let A = (a,b) be any point in a coordinate plane distinct from the origin O. Let M be the midpoint of OA, and let P be a point such that MP is perpendicular to OA and the lengths $\overline{MP}$ and $\overline{OM}$ are equal. Determine the coordinates (x,y) of P in terms of a and b. Give all possible solutions.

3. Determine the exact sum of the series

$$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \cdot \quad \cdot \quad \cdot + \frac{1}{98\cdot 99\cdot 100}$$

4. A six pound weight is attached to a four foot nylon cord that is looped over two pegs in the manner shown in the drawing. At B the cord passes through a small loop in its end. The two pegs A and C are one foot apart and are on the same level. When the weight is released the system obtains an equilibrium position. Determine angle ABC for this equilibrium position, and verify your answer. (Your verification should assume that friction and the weight of the cord are both negligible, and that the tension throughout the cord is a constant six pounds.)



5. The four corners of a rectangle have the property that when they are taken three at a time, they determine triangles all of which have the same perimeter. We will consider whether a set of five points can have this property.

Let $S = \{p_1, p_2, p_3, p_4, p_5\}$ be a set of five points. For each i and j, let d_{ij} denote the distance from p_i to p_j. Suppose that S has the property that all triangles with vertices in S have the same perimeter.

(a) Prove that d_{ij} must be the same for every pair (i,j) with $i \neq j$.

(b) Can such a five-element set be found in three dimensional space? Justify your answer.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America

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