

TWENTY-FIFTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 9, 1981

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Carefully record your six digit MMPC code number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all the problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem in the upper righthand corner of each additional sheet.
5. If you are unable to completely solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you given an explanation of why the generalization might be true.
7. The competition rules do not allow anyone to answer any questions. The use of notes, reference material, computational aids, or any other aid is prohibited. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.
8. You may now open the test booklet and begin.

Score

	1	2	3	4	5	TOTAL
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1. A canoeist is paddling upstream in a river when she passes a log floating downstream. She continues upstream for awhile, paddling at a constant rate. She then turns around and goes downstream and paddles twice as fast. She catches up to the same log two hours after she passed it. How long did she paddle upstream?

2. Let $g(x) = 1 - \frac{1}{x}$ and define $g_1(x) = g(x)$ and $g_{n+1}(x) = g(g_n(x))$ for $n = 1, 2, 3, \dots$. Evaluate $g_3(3)$ and $g_{1982}(1982)$.

3. Let Q denote quadrilateral $ABCD$ where diagonals AC and ED intersect. If each diagonal bisects the area of Q prove that Q must be a parallelogram.

4. Given that a_1, a_2, \dots, a_7 and b_1, b_2, \dots, b_7 are two arrangements of the same seven integers, prove that the product $(a_1 - b_1)(a_2 - b_2) \dots (a_7 - b_7)$ is always even.

5. In analyzing the pecking order in a finite flock of chickens we observe that for any two chickens exactly one pecks the other. We decide to call chicken K a king provided that for any other chicken X , K pecks X or K pecks a third chicken Y who in turn pecks X . Prove that every such flock of chickens has at least one king. Must the king be unique?

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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