

TWENTY-FOURTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 10, 1980

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Carefully record your six digit MMPC code number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. PLEASE DO NOT WRITE YOUR NAME ON THIS BOOKLET.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all the problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, indicate this on the exam page and write your identification number and the problem number in the upper righthand corner of each additional sheet.
5. If you are unable to completely solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a nontrivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. The competition rules do not allow anyone to answer any questions. The use of notes, reference material, computational aids, or any other aid is prohibited. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.
8. You may now open the test booklet and begin.

Score _____

	1	2	3	4	5	TOTAL
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1. On an escalator moving at constant speed a woman in a hurry walks up 9 steps as she travels from one floor to the next higher. A man in an even greater hurry runs 25 steps up the same escalator and reaches the top in half the time the woman took. How many steps does the escalator have between the two floors?

2. A circle inscribed in a right triangle divides the hypotenuse at its point of contact into segments of length x and y . Find (in terms of x and y)
- a) the area of the right triangle b) the diameter of the circle.

4. Consider the positive integers $49, 4489, 444889, \dots$ etc., in which each number after the first is obtained by inserting the digits 4 and 8 (in that order) into the middle of the preceding number. Prove that all these numbers are perfect squares.

5. The sequence $\{100, 55, 45, 10, 35\}$ has five terms, and each term starting with the third is the difference of the preceding two. A sequence terminates when the next term would be negative (since $10 - 35 = -25$, the above example terminates with 35). Zero terms are permitted. Find a positive integer B such that the sequence $\{100, B, \dots\}$ formed as indicated has the maximum number of terms. Generalize by showing how to find a positive integer B that will maximize the length of the sequence $\{A, B, \dots\}$ formed as above, where A is any given positive integer.

The Michigan Mathematics Prize Competition is an activity of the Michigan
Section of the Mathematical Association of America.

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Central Michigan University

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