

TWENTY-THIRD ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by the

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 12, 1979

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Carefully record your six digit student number in the upper righthand corner of this page. This is the only way to identify you with this test booklet. Please do not write your name on this booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and the problem number in the upper righthand corner of each sheet.
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. The competition rules do not allow your supervisor to answer any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score

1

2

3

4

5

TOTAL

1. Solve for  $x$  and  $y$ , if  $\frac{1}{x^2} + \frac{1}{xy} = \frac{1}{9}$  and  $\frac{1}{y^2} + \frac{1}{xy} = \frac{1}{16}$ .

2. Find positive integers  $p$  and  $q$ , with  $q$  as small as possible, such that

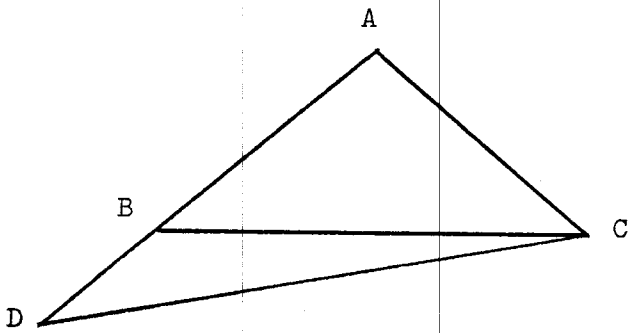
$$\frac{7}{10} < \frac{p}{q} < \frac{11}{15} .$$

3. Define  $a_1 = 2$  and  $a_{n+1} = a_n^2 - a_n + 1$  for all positive integers  $n$ .

If  $i > j$ , prove that  $a_i$  and  $a_j$  have no common prime factor.

4. A number of points are given in the interior of a triangle. Connect these points, as well as the vertices of the triangle, by segments that do not cross each other until the interior is subdivided into smaller disjoint regions that are all triangles. It is required that each of the given points is always a vertex of any triangle containing it. Prove that the number of these smaller triangular regions is always odd.

5. In triangle ABC let  $\angle ABC = \angle ACB = 40^\circ$ . AB is extended to D such that  $AD = BC$ . Prove that  $\angle BCD = 10^\circ$ .



The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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J. K. Bidwell  
Central Michigan University

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