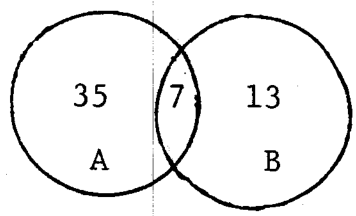
23RD ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Part I Answers

No.	Ans.	Comments
1.	D	
2.	E	$r = \sqrt{2}$, Sum $= a_1(1 + \sqrt{2} + 2 + 2\sqrt{2} + 4) = a_1(7 + 3\sqrt{2})$
3.	A	
4.	C	
5.	C	abc/phd articles are produced each person-hour.
6.	D	
7.	E	$a_k = a_{6m+k}$. $a_1 + \cdots + a_6 = 0$. $a_1 + \cdots + a_{100} = a_{97} + \cdots + a_{100} = 1 + 2 + 1 + -1 = 3$.
8.	A	
9.	E	
10.	A	Let $\alpha = 1 + \theta$ and apply half-angle identity.
11.	A	There are ${}_{20}C_2 = 190$ pairs and 19 adjacent pairs.
12.	C	
13.	B	Let f = father, s = son, $s = f^{-1}$, y = that man, x = me, then $f(y) = sf(x) = x$ or $y = s(x)$.
14.	C	$P(-1) = 0$, therefore, $x - (-1)$ is a factor.
15.	C	$1/6 = 1/10 + 1/x$.
16.	D	A positive fraction implies x is positive so $x - \|x\| = 0$.
17.	A	
18.	D	If $F = 90^\circ$, then $\sin G = \cos H$, $\sin^2 90^\circ + \cos^2 H + \sin^2 H = 1 + 1 = 2$
19.	C	If $x^2 \leq 1$, $y^2 = 1 - x^2$ (circle); if $x^2 \geq 1$, $y^2 = x^2 - 1$ (hyperbola).

20.	D	$\overline{A \cup (\bar{B} \cap \bar{A})} = \bar{A} \cap \overline{(\bar{B} \cap \bar{A})} = \bar{A} \cap (B \cup A) = (\bar{A} \cap B) \cup (\bar{A} \cap A) = \bar{A} \cap B$
21.	D	PQ is mean proportional between AQ and QB.
22.	E	
23.	A	$\Delta MBX = 1/4 \Delta CMB$, by symmetry $r = 12$.
24.	C	Inner circle radius is $R/\sqrt{2}$, so area is 1/2 as large.
25.	C	$2\log_{16} A = \log_4 A$. Thus $\log_{16} A = 1.5$.
26.	E	$\frac{x+1}{x} - \frac{x}{x+1} = \frac{1}{x} + 1 - \frac{x}{x+1} = \frac{1}{x} + \frac{1}{x+1}$
27.	E	35, see diagram. 
28.	A	
29.	B	$m(\overset{\frown}{A'C'}) < 180^\circ$, therefore $\angle C'B'A' < 90^\circ$.
30.	B	y is maximal when $(x - 2)^2$ is minimal.
31.	B	$\Delta ABD \sim \Delta DBC$
32.	A	$a^{\log_a 5} = 5$.
33.	C	$\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2 - 1}$ Middle terms add to zero. Twice the sum $= 1 + 1/2 - 1/20 - 1/21$.
34.	C	$x + y + z = \text{Area } \Delta ABC$.
35.	E	$6A = 6\log_{64} 289 = \log_2 289 = 2\log_2 17 = 2B$.
36.	C	If $x = 10^3$, then we have $x^7 \div (x + 2)$. Remainder theorem says polynomial remainder is $(-2)^7 = -128$. Actual remainder must be $1002 - 128$.
37.	B	
38.	E	Since $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2$, $x^2 = 2$.
39.	A	$.8181\ldots = 9/11$, Thus $pq = 9 \cdot 11 \cdot k^2$ for integer k.
40.	B	Square the fraction and reduce.