

TWENTY-SECOND ANNUAL  
MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by  
MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 6, 1978

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Record, in the upper righthand corner of this page, your six digit student number. This is the only way to identify this test booklet with your name. Please do not write your name on this booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions. To receive full credit for a problem, you are expected to justify your answer.
3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper righthand corner of each sheet.
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score

1

2

3

4

5

Total

1. A rectangle  $ABCD$  is cut from a piece of paper and folded along a straight line so that the diagonally opposite vertices  $A$  and  $C$  coincide. Find the length of the resulting crease in terms of the length ( $l$ ) and width ( $w$ ) of the rectangle. (Justify your answer.)

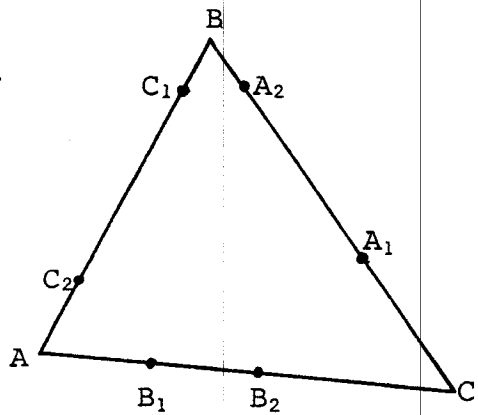
2. The residents of Andromeda use only bills of denominations \$3 and \$5 . All payments are made exactly, with no change given. What whole-dollar payments are not possible? (Justify your answer.)

3. A set consists of 21 objects with (positive) weights  $w_1, w_2, w_3, \dots, w_{21}$ . Whenever any subset of 10 objects is selected, then there is a subset consisting of either 10 or 11 of the remaining objects such that the two subsets have equal total weights. Find all possible weights for the objects. (Justify your answer.)

4. Let  $P(x) = x^3 + x^2 - 1$  and  $Q(x) = x^3 - x - 1$ .

Given that  $r$  and  $s$  are two distinct solutions of  $P(x) = 0$ , prove that  $rs$  is a solution of  $Q(x) = 0$ .

5.



Given:  $\triangle ABC$  with points  $A_1$  and  $A_2$  on  $\overline{BC}$ ,  $B_1$  and  $B_2$  on  $\overline{CA}$ , and  $C_1$  and  $C_2$  on  $\overline{AB}$ .  
 $A_1, A_2, B_1, B_2$  are on a circle,  
 $B_1, B_2, C_1, C_2$  are on a circle,  
and  $C_1, C_2, A_1, A_2$  are on a circle. The centers of these circles lie in the interior of the triangle.

Prove: All six points  $A_1, A_2, B_1, B_2, C_1, C_2$  are on a circle.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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