22nd ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Part I Answers

No.	Ans.	Comments. (Many problems can be solved in several ways.)
1.	C	One of the 2 parts is a similar triangle with half the altitude and half the base, and so one quarter the area.
2.	A	
3.	B	$2(4\cdot 6) + 2(6\cdot 8) + 2(4\cdot 8)$
4.	E	
5.	C	$(1 - .3)(1 - .1) = .63$
6.	A	$2(x^2 - \frac{5}{2}x + 4) = 2(x - r)(x - s) = 2(x^2 - (r + s)x + rs)$
7.	B	113
8.	C	$x = -1 \pm \sqrt{3}\, i$
9.	D	$V = (\frac{S}{6})^{\frac{3}{2}}$, since $V = e^3$ and $S = 6e^2$.
10.	C	
11.	E	Exterior angles measure $\frac{6}{5}\pi$.
12.	B	$45 = 36 + 9$
13.	B	
14.	E	1001000
15.	D	$\frac{1}{2}(10\cdot 9)$
16.	A	
17.	B	$\sec^2\theta + \csc^2\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = \frac{4}{(2\sin\theta\cos\theta)^2}$
18.	C	
19.	D	$= 0$ if x is odd, 2 if x is even
20.	A	Set $x = y = 1$.

21. D $\frac{x}{x-2} > \frac{1}{2}$. So $\frac{2x - (x - 2)}{2(x - 2)} > 0$

22. A

23. B Two vertices of a maximal triangle coincide with adjacent vertices of the square, and the third vertex lies on the opposite side. Otherwise, a triangle with the same base and a larger altitude can be constructed.

24. E The sum of the altitudes of the two triangles remains the same.

25. B $(6 \cdot 5 \cdot 4)/6^3$

26. D

27. E 5 is a divisor of 18 of these factors, 5^2 a divisor of 4 of them.

28. A $3 = 4^{\log_4 3}$

29. C

30. A

31. D $\overline{YW}$ is congruent to $\overline{OX}$.

32. C Each side must have measure no greater than the sum of the other two.

33. C If $b^3 \leq 10^6$, then $b \leq 100$. If, for some integer a, $a^2 = b^3$, then $b\sqrt{b} = a$, and so $\sqrt{b}$ is an integer.

34. C No more than one E: 60 ; 2 E's: 12 ; 3 E's: 1 .

35. C

36 B $150/(\frac{100}{30} + \frac{50}{60})$

37. C $G = \log\left[\left(\frac{1+x}{1-x}\right)^3\right]$

38. D $r = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ and its conjugate $\overline{r}$ are 4th roots of -1. Hence $(x - r)(x - \overline{r})$ is a factor.

39. E $\triangle ABC$ is equilateral. In $\triangle ADC$ $2(\text{length of } \overline{DC})^2 = x^2$.

40. C $90L + x = 91(L + 1)$