

TWENTY-SECOND ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America with the assistance of Michigan Colleges and Universities, Professional Organizations, and Industries.

PART 1

October 18, 1978

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Your answer sheet will be graded by machine. Please read and follow carefully the instructions printed on the sheet. Check to insure that your six-digit student number has been recorded correctly. Do not make calculations on the answer sheet.
2. Do as many problems as you can in the 100 minutes allowed. When the proctor requests you to stop, please cease to work immediately and turn in your answer sheet.
3. Essentially all of the problems require some figuring. Do not be hasty in your judgments. For each problem you should work out ideas on scratch paper before selecting the answer.
4. The 40 problems of this examination are intended to sample many of the topics in the secondary mathematics curriculum. You may be unfamiliar with some of the topics covered in the earlier problems and quite possibly will find a number of problems which are easier for you toward the end. Usually a score of about 20 or more will allow you to become a finalist and write the second exam.
5. In each of the questions five different possible responses are proposed. In some cases the fifth alternative is listed "E, none of these". In such cases if you believe none of the first four alternatives to be correct, mark E.
6. Your score on the test will be the number correct. You are advised to guess an answer in those cases where you cannot determine the right answer or are able to eliminate some of the alternatives as impossible.
7. The person supervising this test is not permitted to explain to you the meaning of any question, so do not request your supervisor to break the rules of the competition. The use of books, tables, slide rules or electronic calculators is prohibited. If you have questions concerning the instructions ask them now.

22nd ANNUAL MICHIGAN MATHEMATICS

PRIZE COMPETITION

1. The line through the midpoints of two sides of a triangle divides the area into two parts. The ratio of the areas of the two parts is

A. 2:1 B. $2\sqrt{2}:1$ C. 3:1 D. $2\sqrt{3}:1$ E. none of these

2. Which of the following cross sections could not have been cut from a solid right circular cone?



A. I only B. II only C. I and III only D. I and IV only
E. I, III, and IV only

3. A cheese fills a box 6 cm x 8 cm x 10 cm. The cheese is coated with a thin layer of wax and cut into unit cubes (1 cm x 1 cm x 1 cm). The number of cubes having wax on exactly one side is

A. 104 B. 208 C. 286 D. 376 E. 480

4. If $a = 0$, then $\frac{1}{a}$

A. is 0 B. is 1 C. is $+\infty$ D. is $-\infty$ E. does not exist

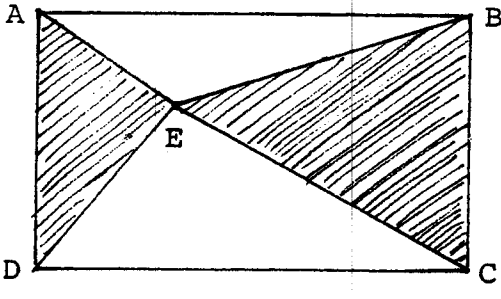
5. Successive discounts of 30% and 10% on an item are equivalent to a simple discount of

A. 40% B. 35% C. 37% D. $33\frac{1}{3}\%$ E. 25%

6. Let S and P denote, respectively, the sum and the product of the roots of $2x^2 - 5x + 8 = 0$. Then SP equals
- A. 10 B. -10 C. 40 D. -40 E. $5 \pm i\sqrt{39}$
7. How many primes lie between 110 and 120 ?
- A. 0 B. 1 C. 2 D. 3 E. 4
8. If $(x + 2)^{-1} = x^{-1} + 2^{-1}$, then
- A. x can be any real number B. $x = 0$
C. x is a complex number D. $x = 1$ E. $x = -2$
9. The total surface area of one cube is 64 times that of another cube. The ratio of their volumes is
- A. 64 to 1 B. 8 to 1 C. 4 to 1 D. 512 to 1
E. none of these
10. A circle has diameter 24, and arc AB subtends a central angle of 60° . The length of arc AB is
- A. 2 B. 12 C. 4π D. 24π E. none of these
11. Each exterior angle of a regular decagon (10-sided polygon), in radians, has measure
- A. $\pi/20$ B. $\pi/10$ C. $3\pi/10$ D. $2\pi/5$ E. none of these

12. Which of the following represents a possible length of the hypotenuse of a right triangle whose legs are integers?
A. $\sqrt{44}$ B. $\sqrt{45}$ C. $\sqrt{46}$ D. $\sqrt{47}$ E. $\sqrt{48}$
13. If logarithms are to base ten, then $\log 3^2 + \log 4^2 + \log 5^2$ equals
A. $\log 50$ B. $2 \log 60$ C. $2 \log 12$ D. $60 \log 2$
E. none of these
14. The number of zeros necessary when 72 is written in base two is
A. 1 B. 2 C. 3 D. 4 E. 5
15. A country has exactly ten cities, no three on the same straight line. Straight roads joining each pair of cities are constructed. The number of such roads is
A. 5 B. 9 C. 17 D. 45 E. 90
16. For sets A, B, and complements \bar{A} , \bar{B} , the set $(\overline{\bar{A} \cup B}) \cup B$ is the same as
A. $A \cup B$ B. $\bar{\phi}$ C. B D. $A \cap B$ E. none of these
17. $4(\sec^2 \theta + \csc^2 \theta)$ is identical to
A. 4 B. $16/\sin^2 2\theta$ C. $2 \sin \theta \cos \theta$ D. $2 \tan \theta$
E. none of these

18. Of the following, the inequality not true for all real numbers x and y is
- A. $|x + y| \leq |x| + |y|$ B. $|x - y| \leq |x| + |y|$
C. $|x - y| \leq |x| - |y|$ D. $\sqrt{x^2} \geq x$ E. $xy \leq |x| |y|$
19. If x is an integer, $(-1)^{x^2} + (+1)^{x^3}$
- A. is always negative B. is always positive
C. sometimes is positive and sometimes negative
D. can have one of only two values E. is meaningless
20. The sum of the coefficients in the expansion of $(x^2 - 2xy + y^2)^{17}$ is
- A. 0 B. 17 C. 34 D. 2^{17} E. 2^{34}
21. A certain fraction is greater than $\frac{1}{2}$. Its denominator is two less than its numerator. If x represents the numerator, then
- A. $x > 2$ B. $x < -2$ C. $x > 2$ and $x < -2$
D. $x > 2$ or $x < -2$ E. $x = 3$
22. If x is real, the minimum value of $(x - 2)^2 - 16$ is
- A. -16 B. 0 C. -12 D. 4 E. 20
23. The area of the largest triangle that can be put inside a 4-by-4 square is
- A. $4\sqrt{2}$ B. 8 C. $8\sqrt{2}$ D. 10 E. 16

24.  In the figure at the left, ABCD is a rectangle. Point E is closer to \overline{AD} than to \overline{BC} . If we move point E towards \overline{BC} , the area covered by the shaded parts will

- A. increase B. increase, then decrease C. decrease
D. decrease, then increase E. remain the same

25. When 3 dice are tossed, the probability that all the faces showing are different is

A. $\frac{2}{3}$ B. $\frac{5}{9}$ C. $\frac{1}{3}$ D. $\frac{20}{216}$ E. none of these

26. Given $2^{-3k} = b$ and $2^{-k} = 8b$, then k is

A. $-\frac{2}{3}$ B. $-\frac{3}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$ E. none of these

27. How many consecutive zeros stand at the end of the number $N = (15)(16)(17) \dots (101)$?

A. 8 B. 10 C. 18 D. 19 E. 22

28. The result of raising 3 to the power $(\log_4 8)/(\log_4 3)$ is

A. 8 B. 3 C. 4 D. $8/3$ E. none of these

29. If $\sqrt{x^2 + 2xy + y^2} = x + y$, then

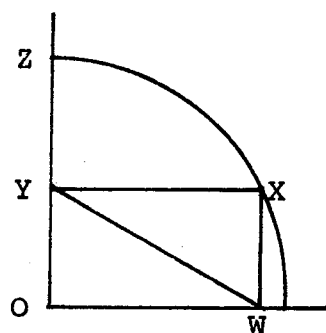
A. $x = y$ B. $x \geq y$ C. $x \geq -y$ D. $x \geq 0$ and $y \geq 0$
E. none of these

30. If $\frac{x^2 - 4x + 4}{x^3 - 3x^2 + 4} = \frac{A}{x + 1} + \frac{B}{(x - 2)^2}$, then A and B are,

respectively,

- A. 1 and 0 B. 1 and 2 C. 1 and 5
D. 1 and 7 E. none of these

31. The figure at the right shows a rectangle inscribed in a quarter-circle. If the measure of \overline{OZ} is 5, what is the measure of \overline{YW} ?



- A. $3\sqrt{2}$ B. $3\frac{1}{2}$ C. 4 D. 5
E. Insufficient information given

32. A non-degenerate triangle has sides of 1 , $1 - c$, and $1 - 2c$. c must be in which interval?

- A. $(\frac{1}{3}, 1)$ B. $(-1, -\frac{1}{3})$ C. $(-1, \frac{1}{3})$ D. $(-\frac{1}{3}, 1)$

E. none of these

33. Among the first million positive integers, the number of perfect squares which are also perfect cubes is

- A. 1 B. 2 C. 10 D. 100 E. none of these

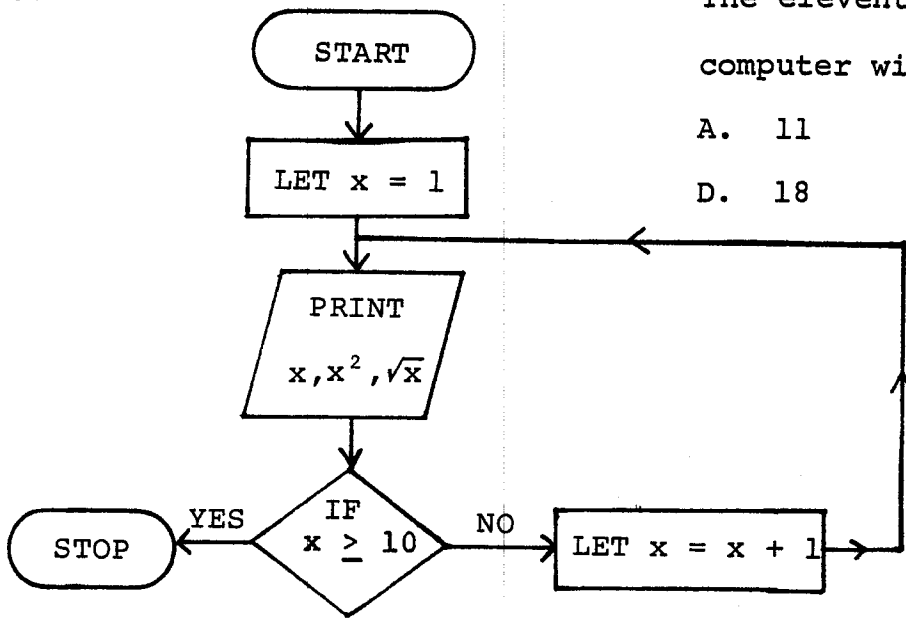
34. EBE, BEE, and NTW are different 3 letter strings constructed from the word BETWEEN. The total number of such strings is

- A. 35 B. 70 C. 73 D. 135 E. 210

35.

The eleventh number printed by the computer will be

- A. 11 B. 121 C. 16
D. 18 E. 4



36. A car travels 30 km/hour going uphill and 60 km/hour going downhill. What is its average speed, in km per hour, if it goes 100 km uphill and then 50 km downhill?

- A. 40 B. 36 C. $33\frac{1}{3}$ D. 45 E. $48\frac{1}{2}$

37. Let $F = \log \frac{1+x}{1-x}$. Form a new function G by replacing each x in F by $\frac{3x+x^3}{1+3x^2}$ and simplify. The simplified G is equal to

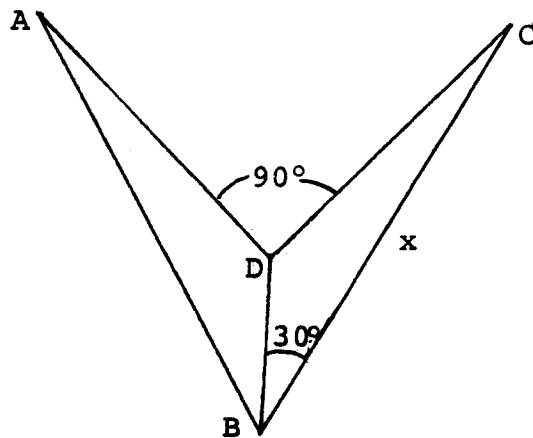
- A. $-F$ B. F C. $3F$ D. F^3 E. $F^3 - F$

38. A factor of $x^4 + 1$ is

- A. $x - i$ B. $x + i$ C. $x^2 - x - 1$ D. $x^2 - x\sqrt{2} + 1$
E. none of these

39. In the diagram, if $\triangle ABD$ is congruent to $\triangle CBD$, what is the length of \overline{DC} ?

- A. $x\sqrt{3}/5$
- B. $2x/5$
- C. $x/3$
- D. $2x\sqrt{2}/3$
- E. $x\sqrt{2}/2$



40. A student has taken L tests and has a 90% average. What score must the student get on the next test to raise the average to 91% ?

- A. $L + 2$
- B. $L - 2$
- C. $L + 91$
- D. $L - 91$
- E. $2L + 1$