21st ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Part I Answers

No.	Ans.	Comments. (Many problems can be solved in several ways.)
1.	A	$(A\cap B)\cup(B\cap C)\subseteq B\subseteq A\cup B$
2.	D	$x^4 - 9x^2 + 20 = (x^2 - 5)(x + 2)(x - 2)$
3.	A	$\frac{1}{2}(1 + \frac{1}{10})a(1 - \frac{1}{10})b = \frac{99}{100}(\frac{1}{2}ab)$
4.	E	4 hens lay 8 eggs in 3 days.
5.	A	$18 + 2\cdot\frac{18}{3} + 2\cdot\frac{18}{9} + \cdots = -18 + 36(1 + \frac{1}{3} + \frac{1}{9} + \cdots) = -18 + 36\cdot\frac{3}{2}$
6.	A	$10(100 - x) - 25x = 510$
7.	D	Circle and parabola intersect at (0,0) and (1,1).
8.	D	Distant at least 3 units, and no more than 7, from the number 1 on the x axis.
9.	E	$597 = 5\cdot119 + 2 = 5(5\cdot23 + 4) + 2 = 23\cdot5^2 + 4\cdot5 + 2$
10.	B	$\log 10 - \log 2 = \log 5$
11.	C	$a = -3x^2$
12.	B	$(\frac{1}{2})^n R$ left after nth year. $(\frac{1}{2})^n R \le \frac{1}{1000}R$ if, and only if, $2^n \ge 1000$.
13.	B	$16 + 9 + 4 + 1$
14.	E	
15.	C	
16.	B	
17.	E	
18.	C	
19.	D	$3(x^2 + 3x - \frac{2}{3}) = 3(x - r)(x - s)$
20.	C	
21.	C	Draw the radius containing the two centers and the common point of tangency. The segment between the centers is the diagonal of a square of side r. Hence $\sqrt{2}r + r = R$, and so $r = \frac{\sqrt{2}}{\sqrt{2}+1}$.
22.	E	

23. A $x^2 - 5x + 7 \geq 1$ where the parabola $y = x^2 - 5x + 6 = (x - 3)(x - 2)$ is above the x axis.

24. C Draw a segment joining two consecutive dots to divide the triangle into two triangles with Base 1. One has altitude 1, the other 2.

25. E $-1 + 50(-2)$

26. B Draw perpendiculars from the center of the square to the upper and right hand sides. Because of congruent right triangles formed, the area of the small square formed equals the area of the common quadrilateral.

27. E $(51380 + 7)^{453} = k(51380) + 7^{453}$, by the Binomial Theorem. Since $7^4 = 2401$, the units digit in $7^1, 7^5, 7^9, \ldots$ is 7. $453 = 4 \cdot 113 + 1$.

28. A $2(x - 5 + 3i)(x - 5 - 3i) = 2[(x - 5)^2 - (3i)^2] = 2x^2 - 20x + 68$.

29. D Let $x = RM$. Apply the law of cosines to triangle PRM to get $\frac{49}{4} = x^2 + 49 - 14x\cos R$ and to PRQ to get $16 = 4x^2 + 49 - 28x\cos R$. Multiply the first equation by 2 and subtract from the second to get a quadratic equation in x.

30. E $10^{20} = 100^{10} = (98 + 2)^{10} = k \cdot 98 + 2^{10}$. So the remainder is that obtained when 1024 is divided by 98, namely, 44.

31. D Let P be the other vertex of the rhombus on $\overline{AB}$, Q the vertex on $\overline{BC}$, and R the other vertex on $\overline{AC}$. Let x be the measure of a side of the rhombus. Triangles PBQ and QCR are similar. Hence $\frac{6 - x}{x} = \frac{x}{12 - x}$

32. B

33. A $3 \cdot 10$ is the total number of line ends in the network. Each line has 2 ends.

34. B $(-2)^{12} - 9$, by the Remainder Theorem.

35. D Triangle PQR is a right triangle. $PR = PQ\cos P$ and $QR = PQ\sin P$. So $PR + QR = PQ(\cos P + \sin P) = PQ[\sqrt{2}\sin(P + \frac{\pi}{4})]$.

36. A

37. B The equilateral triangle has area $\sqrt{3}r^2$. Each of the 3 circular sectors inside the triangle has area $\frac{1}{6}\pi r^2$.

38. C Draw the radii to the three vertices. Each of the 3 congruent triangles formed has area $\frac{1}{2} \cdot 7(7\sin 60^\circ)$.

39. C $(2n + 1)^2 = 4(n^2 + n) + 1$

40. D