

TWENTIETH ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by
MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 8, 1976

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Record, in the upper righthand corner of this page, your six digit student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.
3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper righthand corner of each sheet.
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score _____
 1 2 3 4 5 Total

1. The total cost of 1 football, 3 tennis balls and 7 golf balls is \$14, while that of 1 football, 4 tennis balls and 10 golf balls is \$17. If one has \$20 to spend, is this sufficient to buy
- a) 3 footballs and 2 tennis balls?
 - b) 2 footballs and 3 tennis balls?

2. Let \overline{AB} and \overline{CD} be two chords in a circle intersecting at a point P (inside the circle).
- a) Prove that $AP \cdot PB = CP \cdot PD$.
- b) If \overline{AB} is perpendicular to \overline{CD} and the length of \overline{AP} is 2, the length of \overline{PB} is 6, and the length of \overline{PD} is 3, find the radius of the circle.

3. A polynomial $P(x)$ of degree greater than one has the remainder 2 when divided by $x-2$ and the remainder 3 when divided by $x-3$. Find the remainder when $P(x)$ is divided by x^2-5x+6 .

4. Let $x_1 = 2$ and $x_{n+1} = x_n + (3n+2)$ for all n greater than or equal to one.
- Find a formula expressing x_n as a function of n .
 - Prove your result.

5. The point M is the midpoint of side \overline{BC} of a triangle ABC .

a) Prove that $AM \leq \frac{1}{2}AB + \frac{1}{2}AC$.

b) A fly takes off from a certain point and flies a total distance of 4 meters, returning to the starting point. Explain why the fly never gets outside of some sphere with a radius of one meter.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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ACKNOWLEDGMENTS

The following industries and professional organizations have provided generous financial support to this competition.

Michigan Bell Telephone
Burroughs Corporation
Kuhlman Corporation
Michigan Council of Teachers of Mathematics

The Michigan Association of Secondary School Principals has placed this competition on the Approved List of Michigan Contests and Activities.