20th ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Part I Answers

No.	Ans.	Comments. (Many problems can be solved in several ways.)
1.	C	
2.	D	Sum is $5\cdot 1 + 3^2 + 9\cdot 3 = 41$.
3.	B	$9x^2 + 30x + 25 = (3x + 5)^2$.
4.	A	Multiply by 6 to get $6x - 9 + 2x - 8 = 0$.
5.	D	Common denominator is $(x-y)(x+y)$, numerator is $x + y + x - y = 2x$.
6.	E	$2^{-10} = 1/1024 \approx .0009765625$.
7.	B	The hypotenuse is a diameter. Thus the radius is $\sqrt{2}/2$.
8.	A	Collecting the real part and imaginary part yields the two equations $2x - 3y = 0$ and $y + 2 = 0$. Thus $y = -2$, $x = -3$.
9.	C	
10.	A	BD = 8. Triangle ABC has same area as BDC which is $\frac{1}{2}\cdot 6\cdot 8$.
11.	B	
12.	E	Substituting $x = -1$ yields $81a + 9b + c = 0$. $x = 5$ produces the same result since $(-3)^2 = 3^2$.
13.	E	Draw $\overline{BO}$. Use isosceles triangles ABO and BOE and exterior angles. If angle BAC is α then angle OBE is 2α. From triangle AOE we obtain $\alpha + 2\alpha = 75$.
14.	A	From $3^{2x} = 4$ obtain $2x\log 3 = \log 4 = 2\log 2$.
15.	C	$(x+y)^2 = (x-y)^2 + 4xy = 16 + 8 = 24$. $\lvert x+y\rvert = \sqrt{24}$
16.	E	$V = 3\pi r^3/4$. Doubling r increases volume by a factor of 8.
17.	C	There are 16 ways to succeed, 13 clubs and the 3 other aces.
18.	E	$\log_{10} x = \log_{10} 2^3 - \log 10^2 = \log_{10}(8/100)$ so $x = .08$.
19.	B	$\sin 2\theta = 2\sin\theta\cos\theta$.
20.	D	The remainder upon division by $x+1$ is equal to the value obtained by substituting $x = -1$.
21.	D	The right inequality in $-1 < \frac{x-1}{x+3} < 1$ is satisfied whenever $x > -3$, the left by $x < -3$ or $x > -1$. Needing both satisfied, take the intersection.

22.	A	$f(1) = 1$ yields $a = 4$.
23.	B	Sum is $n(a + d(n-1)/2) = (n/2)(2a + (n-1)d)$ where $a = 5$ and $d = 11$.
24.	E	$2x^2 + 7x - 22 = (2x + 11)(x - 2)$ so roots are 2 and $-11/2$.
25.	C	Denote the number by a. Then $1000a - 10a = 1422$. Solve for a and reduce.
26.	B	$20 = 5 \cdot 2^2$. One need only count the number of 5's appearing.
27.	C	
28.	E	$AD = 5\sin 40^\circ$ $\quad DC = AD\tan 60^\circ$
29.	C	
30.	A	Let $PA = x$. Use theorem from geometry to get $x(x + 156) = (x + 12)(x + 12 + 96)$. Solve for x.
31.	E	The line through the two given points has slope 4. Our line must have slope $m = -\frac{1}{4}$. Use point-slope equation.
32.	D	Arc ABC measures $44^\circ + 16^\circ$. Thus arc ADC has length $(300/360) \cdot 2\pi(9)$.
33.	D	By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos C$. Must have $\cos C$ negative, so C is obtuse.
34.	A	There are $6! = 24$ ways to arrange 6 digits. Divide by $3! \cdot 3!$ to take care of repetitions.
35.	B	One person paints 1/24 car per hour.
36.	B	$x^2 + 8x = (x + 4)^2 - 16$. $(x + 4)^2$ is always nonnegative.
37.	D	Let $BM = x$. Use 30°- 60° right triangles. $(2x)^2 = x^2 + 10^2$. Solve for x.
38.	C	Let d denote the distance, b the average speed returning. Then, equating time, $d/r + d/b = 2d/s$. Thus $b = rs/(2r-s)$.
39.	D	Because of integer coefficients, $P(x)$ must have $(x-1)$, $(x-\sqrt{3})$, $(x+\sqrt{3})$, $(x-i)$, and $(x+i)$ as factors.
40.	A	The sum of the roots of a monic polynomial of degree n is the negative of the coefficient of x^{n-1}. Divide by 2 to obtain a monic polynomial. The sum of the roots is 90. There are 10 roots.