Student Number

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NINETEENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 3, 1975

INSTRUCTIONS
(to be read aloud to class by supervisor or proctor)

1. Record, in the upper righthand corner of this page, your six digit student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.

3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.

4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper righthand corner of each sheet.

5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score						
	1	2	3	4	5	Total

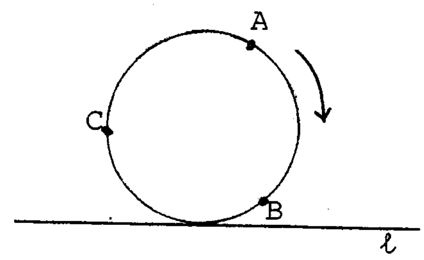
1. a) Given four points in the plane, no three of which lie on the same line, each subset of three points determines the vertices of a triangle. Can all these triangles have equal areas? If so, give an example of four points (in the plane) with this property, and then describe all arrangements of four points (in the plane) which permit this. If no such arrangement exists, prove this.

b) Repeat part a) with "five" replacing "four" throughout.

2. Three people at the base of a long stairway begin a race up the stairs. Person A leaps five steps with each stride (landing on steps 5, 10, 15, etc.). Person B leaps a little more slowly but covers six steps with each stride. Person C leaps seven steps with each stride. A picture taken near the end of the race shows all three landing simultaneously, with Person A twenty-one steps from the top, Person B seven steps from the top, and Person C one step from the top. How many steps are there in the stairway? If you can find more than one answer, do so. Justify your answer.

3. Let S denote the sum of an infinite geometric series. Suppose the sum of the squares of the terms is $2S$, and that of the cubes is $64S/13$. Find the first three terms of the original series.

4. A, B, and C are three equally spaced points on a circular hoop. Prove that as the hoop rolls along the horizontal line ℓ, the sum of the distances of the points A, B, and C above line ℓ is constant.



5. A set of n numbers $x_1, x_2, x_3, \ldots, x_n$ (where $n>1$) has the property that the k^{th} number (that is, x_k) is removed from the set, the remaining $(n-1)$ numbers have a sum equal to k (the subscript of x_k), and this is true for each $k = 1, 2, 3, \ldots, n$.

a) Solve for these n numbers

b) Find whether at least one of these n numbers can be an integer.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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