

NINETEENTH ANNUAL
MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America with the assistance of Michigan Colleges and Universities, Professional Organizations, and Industries.

PART 1

October 15, 1975

INSTRUCTIONS

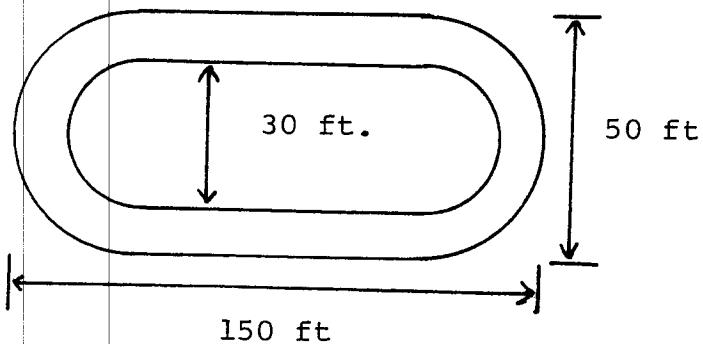
(to be read aloud to class by supervisor or proctor)

1. Your answer sheet will be graded by machine. Please read and follow carefully the instructions printed on the sheet. Check to insure that your six-digit student number has been recorded correctly. Do not make calculations on the answer sheet.
2. Do as many problems as you can in the 100 minutes allowed. When the proctor requests you to stop, please cease work immediately and turn in your answer sheet.
3. Essentially all of the problems require some figuring. Do not be hasty in your judgments. For each problem you should work out ideas on scratch paper before selecting the answer.
4. The first 20 problems of this examination are intended to sample many of the topics in the secondary mathematics curriculum. You may be unfamiliar with some of these topics and quite possibly will find a number of problems which are easier for you distributed throughout the last twenty items. Usually a score of about 20 or more will allow you to become a finalist and write the second exam.
5. In each of the questions five different possible responses are proposed. In some cases the fifth alternative is listed "E none of these". In such cases if you believe none of the first four alternatives to be correct, mark E.
6. Your score on the test will be the number correct. You are advised to guess an answer in those cases where you cannot determine the right answer or are able to eliminate some of the alternatives as impossible.
7. The person supervising this test is not permitted to explain to you the meaning of any question, so do not request your supervisor to break the rules of the competition. The use of slide rules or electronic calculators is prohibited. If you have questions concerning the instructions ask them now.

19th ANNUAL MICHIGAN MATHEMATICS

PRIZE COMPETITION

- 1) A rope of length 300 feet is cut into two pieces, one of which is $\frac{2}{3}$ as long as the other. The shorter piece has length:
 A. $66\frac{2}{3}$ feet B. 100 feet C. 120 feet D. 180 feet
 E. 200 feet
- 2) If $x^2 > 16$ and $x < 1$, then
 A. $x < -4$ B. $-4 < x < 1$ C. $x > 3$ D. $1 < x < 4$
 E. $-1 < x < 0$
- 3) The solution of the equation $\frac{4x}{3} - \frac{2x-1}{4} = \frac{7}{18}$ is
 A. $\frac{1}{2}$ B. 4 C. $\frac{23}{30}$ D. $\frac{1}{3}$ E. $\frac{1}{6}$
- 4) A track for running has the dimensions shown in the diagram. (The turns are semicircles.)
 How much farther is it to go once around the track along the outer edge than along the inner edge?
- A. 20π ft. B. 25π ft. C. 40π ft. D. 40π ft + 100 ft.
 E. 75π ft. + 50 ft.
- 5) Given that $a + b = 5$ and $ab = 2$, then $a^2 + b^2 =$
 A. 25 B. 21 C. 23 D. 29 E. 27



6) Find all x such that $x^2 + x - 6$ is positive.

- A. $x < 0$ B. $-2 < x < 3$ C. $x < -3$ or $x > 2$
D. $x < -2$ or $x > 3$ E. none of these

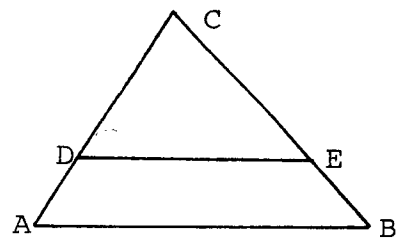
7) Simplification of $2ab - (a-b)^2$ yields

- A. $b^2 - a^2$ B. $4ab - a^2 + b^2$ C. $-(a+b)^2$ D. $a^2 + b^2$
E. none of these

8) In the diagram, \overline{DE} is parallel to \overline{AB} .

If the ratio $CD:DA$ is $3:1$, then $DE:AB$ must be

- A. $2:3$ B. $3:4$ C. $9:16$ D. $3:1$
E. $3:5$

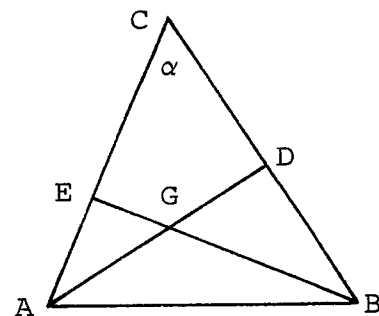


9) A rectangle has area 6 and perimeter 10. What are its dimensions?

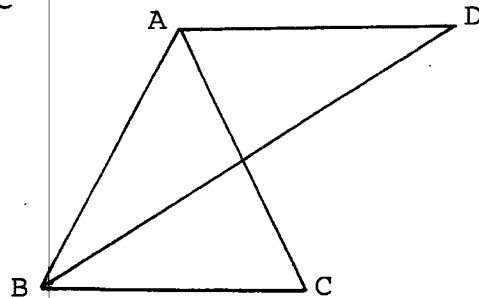
- A. $3/2$ by 4 B. 2 by 3 C. $5/2$ by $12/5$ D. 1 by 6
E. not enough information given

10) Let two altitudes \overline{AD} and \overline{BE} of triangle ABC meet at a point G inside that triangle. If α is the measure of angle ACB in degrees, then angle AGB has degree measure

- A. $90 + \alpha$ B. 2α C. $360 - \alpha$ D. $180 - \alpha$
E. none of these

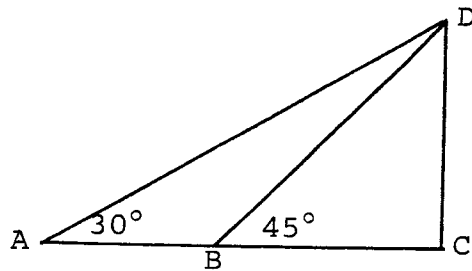


- 11) If $y = x + 1$, then $|y - x| + |x - y|$ equals
A. -1 B. 0 C. 1 D. 2 E. none of these
- 12) Suppose one has a rectangle, and increases the length of each of two sides (parallel to each other) by 10%, and decreases the length of each of the other sides by 10%. Then the area has been
A. increased by 1% B. decreased by 1% C. increased by 1.1%
D. unchanged E. insufficient information
- 13) $\log(x^2) + \log(x^3) + \log(x^4)$ equals
A. $\log(x^2+x^3+x^4)$ B. $x \log 9$ C. $9 \log x$ D. $\log(9x)$
E. $24 \log x$
- 14) Let N be the set of positive integers, P the set of positive even integers, and Q the set of positive integral multiples of 6. If P' and Q' denote the complements of P and Q with respect to N , then the set of positive even integers not divisible by 3 is
A. $P' \cap Q$ B. $P' \cup Q'$ C. $P \cup Q$ D. $P \cap Q'$
E. none of these
- 15) In the figure, \overline{AD} is parallel to \overline{BC} and $AB = AC = AD$. If the angle BAC measures 36° , then the angle ABD measures
A. 28° B. 30° C. 36°
D. 45° E. 48°



- 16) If the numeral 112 (in our usual base ten system) is converted to a base two numeral, how many zeros will appear?
- A. 2 B. 3 C. 4 D. 5 E. 6
- 17) Suppose $1, 50, a_3, \dots$ is an arithmetic progression, and $1, 50, g_3, \dots$ is a geometric progression. Then the sum of the two numbers a_3 and g_3 is
- A. 150 B. 151 C. 2599 D. 2600 E. none of these
- 18) Suppose the top of a rectangular box is found to have area 24 square inches, and the area of one of its sides is 32 square inches, and the area of one of its ends is 12 square inches. What is the height of the box?
- A. 1 inch B. 2 inches C. 3 inches D. 4 inches
E. 6 inches
- 19) For what values of b does the equation $x^2 + bx + 16 = 0$ have different real roots?
- A. $b^2 < 64$ B. $b < 8$ C. $b^2 \geq 64$ D. $b^2 > 64$
E. none of these

- 20) In the figure, the line segment \overline{CD} is perpendicular to \overline{AC} . The angles of elevation to D from A and B measure 30° and 45° respectively. Given that \overline{AB} has length 50, the length of \overline{CD} is



- A. $25(1 + \sqrt{3})$ B. 50 C. $\frac{\sqrt{3}}{2}50$ D. $\frac{\sqrt{2}}{2}50$ E. $\frac{50}{3}\sqrt{3}$

- 21) The expression $(x^9 y^{-6} / z^{-3})^{\frac{1}{3}} / (x^2 z / y)$ when simplified is

- A. x/y B. xy C. $x/(yz^2)$ D. x/y^2 E. $x^2 y$

- 22) Three persons are to be selected from a pool of 20 qualified applicants, 10 of whom are men, and 10 are women. If the three are selected at random what is the probability that all three are men?

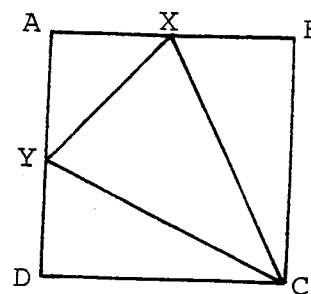
- A. $2/19$ B. $3/20$ C. $3/17$ D. $7/200$ E. $1/3$

- 23) Which is cheapest to the buyer?

- A. a discount of 15%,
B. a discount of 10% followed by a discount of 5% on the new price,
C. a discount of 7.5% followed by a discount of 7.5% on the new price,
D. a discount of 5% followed by a discount of 10% on the new price,
E. all four methods give an equal discount.

- 24) The line l_1 is given by $y + 4x = 7$, and the line l_2 is given by $2y + ax = 23$. If the number a is selected from the integers 1, 2, 3, ..., 10, how many of these ten choices will guarantee that l_1 intersects l_2 ?
- A. 0 B. 1 C. 3 D. 5 E. 9
- 25) If $f(x) = 1/x$ then $f(1/3) - f(-1/3)$ equals
- A. 6 B. $2/3$ C. 3 D. $1/3$ E. 0
- 26) If $i^2 = -1$, of the choices of z below, which satisfies $z^3 = 8$?
- A. -2 B. $2\sqrt{2}$ C. $-1 + i\sqrt{3}$ D. $2i$ E. $1 + i\sqrt{3}$
- 27) A circle is circumscribed about a rectangle of width 6 and length 8. What is the area of the circle?
- A. 9π B. 16π C. 25π D. 48π E. 100π
- 28) If the expression $x^2 + px + 4$ has a minimum value of 3, then p must equal
- A. $\pm 2/3$ B. ± 2 C. $\pm 2\sqrt{7}/3$ D. ± 1 E. ± 8
- 29) How many integers between 1 and 1001 are divisible by 2 or 3?
- A. 600 B. 667 C. 700 D. 724 E. 725
- 30) In the triangle ABC, segment \overline{BC} has length 4, angle BAC measures 40° , and angle ABC measures 60° . The length of segment \overline{AB} is
- A. $\frac{\sin 80^\circ}{4 \sin 40^\circ}$ B. $\frac{4 \sin 40^\circ}{\sin 80^\circ}$ C. $\frac{4 \sin 60^\circ}{\sin 40^\circ}$ D. $\frac{\sin 40^\circ}{4 \sin 80^\circ}$
- E. $\frac{4 \sin 80^\circ}{\sin 40^\circ}$

- 31) If ABCD is a square, and X and Y are the midpoints of \overline{AB} and \overline{AD} respectively, then the ratio of the area of triangle XYZ to that of the square is

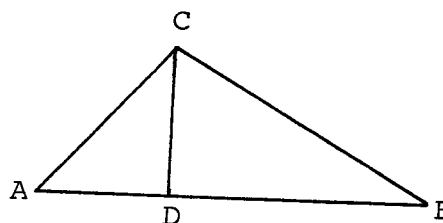


- A. 1:4 B. 5:8 C. 3:8 D. 1:3 E. 1:2

- 32) Find the value of b for which the graphs $y = 3x^2 - x$ and $y = 2x + b$ have exactly one common point.

- A. -1 B. 1 C. -1/2 D. 3/4 E. -3/4

- 33) In the triangle ABC, side \overline{AB} has length 4, side \overline{BC} has length 3, and side \overline{AC} has length 2. If D is chosen so angle CDB has measure 90° , then segment \overline{DB} has length



- A. $\sqrt{3} \frac{3}{2}$ B. $\sqrt{\frac{13}{4}}$ C. $\frac{9}{4}$ D. $\frac{12}{5}$ E. $\frac{21}{8}$

- 34) Which of the following correctly describes the set of points (x,y) which satisfy $(x-y)^2 = 1$?

- A. two lines B. circle C. ellipse D. hyperbola
E. none of these

- 35) If $\log_2(4x-4) = 2$, then $\log_4 x$ equals

- A. 2 B. 4 C. 1 D. 1/2 E. 8

36) Find x if $2^x = 9(2^{-x})$.

- A. 0 B. $\log_{10}(3/2)$ C. $(\log_{10}3)/(\log_{10}2)$ D. $\log_{10}6$
E. $(\log_{10}3)^2$

37) What is the largest integer n such that 2^n is a factor of the product of the first 18 positive integers, namely, $1 \cdot 2 \cdot 3 \cdot \dots \cdot 16 \cdot 17 \cdot 18$?

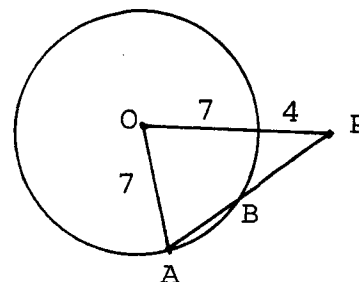
- A. 9 B. 10 C. 12 D. 16 E. 18

38) The distance of the point P from the center O of a circle is 11 inches.

If the radius of the circle is 7 inches,

and A is a point on the circle with the property that the midpoint B of segment

AP also lies on the circle, then AP equals



- A. 7 inches B. 12 inches C. 13 inches D. 14 inches
E. $11\sqrt{2}$ inches

39) The expression $\frac{1 + \cos \theta + \sin \theta}{1 - \cos \theta + \sin \theta}$ is (for all admissible θ) equal to

- A. $\tan \theta$ B. $\sec \theta$ C. $\cos 2\theta$ D. $\frac{1 + \cos \theta}{\sin \theta}$ E. $\frac{1 + \sin \theta}{\cos \theta}$

40) A man has six coins in his pocket. One has heads on both sides, two have tails on both sides, and the other three are normal. He takes one coin from his pocket, lays it on the table, and observes that it shows a head. What is the probability that it is one of the normal coins?

- A. $1/2$ B. $3/5$ C. $2/3$ D. $3/4$ E. 1

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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