The sketch of one solution for each of the Part II problems is given below. Solutions do not include possible generalizations.

1. Let \( S \) be the sum of the 99 terms:
\[
(\sqrt{1} + \sqrt{2})^{-1}, \ (\sqrt{2} + \sqrt{3})^{-1}, \ (\sqrt{3} + \sqrt{4})^{-1}, \ldots, \ (\sqrt{99} + \sqrt{100})^{-1}.
\]
Prove that \( S \) is an integer.

Solution: \( S = (\sqrt{1} + \sqrt{2})^{-1} + (\sqrt{2} + \sqrt{3})^{-1} + \ldots + (\sqrt{99} + \sqrt{100})^{-1} \)
\[
= \frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \ldots + \frac{1}{\sqrt{99} + \sqrt{100}}
\]
Multiplying numerator and denominator of each fraction by the difference of the radicals appearing yields:
\[S = (\sqrt{2} - \sqrt{1}) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3}) + \ldots + (\sqrt{100} - \sqrt{99})\]
Noting that \( \sqrt{n} \) and \( -\sqrt{n} \) occur for all \( n = 2, 3, \ldots, 99 \) permits the simplification:
\[S = -\sqrt{1} + \sqrt{100} = -1 + 10 = 9 \text{ which is an integer.}\]

2. Determine all pairs of positive integers \( x \) and \( y \) for which \( N = x^4 + 4y^4 \) is a prime.

Solution: The question of primeness suggests that we try to represent \( N \) in factored form. Complete the square:
\[
N + 4x^2y^2 = x^4 + 4x^2y^2 + 4y^4 = (x^2 + 2y^2)^2
\]
\[
\therefore N = (x^2 + 2y^2)^2 - 4x^2y^2
\]
\[
= (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)
\]
Thus \( N \) will be prime if and only if
\[
N = x^2 + 2y^2 + 2xy \quad \text{and} \quad 1 = x^2 + 2y^2 - 2xy
\]
The second equation may be rewritten as
\[
(x - y)^2 + y^2 = 1 \Rightarrow x = y = 1 \quad \text{and} \quad N = 5.
\]

Let \( w, x, y, z \) be arbitrary positive real numbers. Prove each inequality:

a) Prove \( xy \leq \left(\frac{x + y}{2}\right)^2 \)

\[
(x - y)^2 \geq 0 \Rightarrow x^2 - 2xy + y^2 \geq 0 \text{ also } x^2 + 2xy + y^2 = (x + y)^2
\]
multiply both sides of the equality by \(-1\) and adding to the inequality yields: \(-4xy \geq -(x + y)^2\) or \(xy \leq \left(\frac{x + y}{2}\right)^2\)

b) Prove \( wxyz \leq \left(\frac{w + x + y + z}{4}\right)^4 \)
From part "a" we know: \( wx \leq \left( \frac{w + x}{2} \right)^2 \) and \( yz \leq \left( \frac{y + z}{2} \right)^2 \)

all terms positive permit the multiplication of these inequalities to yield: \( wxyz \leq \left[ \frac{(w + x)(y + z)}{2} \right]^2 \leq \left[ \frac{(w + x + y + z)}{2} \right]^2 \)

where the second inequality is obtained by an additional application of part "a". The last quantity is precisely the one desired.

c) Prove \( xyz \leq \left( \frac{x + y + z}{3} \right)^3 \)

Use part "b" letting \( w = \frac{x + y + z}{3} \) which yields:

\[
wx = \left[ \frac{x + y + z}{3} \right]^4 = (x + y + z)^4 = w^4
\]

\[
xyz \leq w^4 = xyz \leq w^3 = \left( \frac{x + y + z}{3} \right)^3 \text{ where the division by } w \text{ is permitted since } w > 0.
\]

4. Solution: To prove the three lines are concurrent we shall prove that the point \( Q \) which is the intersection of \( P_1P_9 \) and \( P_4P_{12} \) is the same distance from \( P_{1P_{12}} \) as the line \( P_2P_{11} \) which is parallel to \( P_{1P_{12}} \) and on the same side as \( Q \).

Without loss of generality, let \( P_{1P_{12}} = 2 \text{ units} \) and \( R \) the point where the bisector of \( \angle P_{1QP_{12}} \) intersects \( P_{1P_{12}} \).

Then since \( \triangle P_{1QP_{12}} \) is isosceles with \( 45^\circ \) base angles (inscribed angles), it is easily shown that \( QR \perp P_{1P_{12}} \) and \( QR = 1 \).

Now \( P_{1P_{12}} \) is parallel to \( P_{2P_{11}} \) since the alternate interior angles \( \angle P_{2P_{12}P_{1}} \) and \( \angle P_{1P_{12}P_{2}} \) are congruent. Let \( P_{12}S \) be perpendicular to \( P_{11}P_{2} \) then \( \triangle P_{12P_{11}S} \) is a \( 30^\circ-60^\circ-90^\circ \) triangle with hypotenuse of 2 units. Then \( P_{12}S \) (the distance from \( P_{12P_{1}} \) to \( P_{2P_{11}} \)) is also \( 1 \) unit.

5. Two very busy men, \( A \) and \( B \), who wish to confer, agree to appear at a designated place on a certain day, but no earlier than noon and no later than 12:15 p.m. If necessary, \( A \) will wait 6 minutes for \( B \) to arrive, while \( B \) will wait 9 minutes for \( A \) to arrive but neither can stay past 12:15 p.m. Express as a percent their chance of meeting.

Solution: Let \( A \) arrive \( x \) minutes past noon and \( B \) at \( y \) minutes past noon. Then \( 0 \leq x \leq 15 \) and \( 0 \leq y \leq 15 \).

If \( A \) arrives first or both arrive simultaneously, then \( y \geq x \) and \( 0 \leq y - x \leq 6 \) if the men are to meet (Event I). Similarly if \( B \) arrives first then \( x > y \) and \( 0 < x - y \leq 9 \) (Event II). The probability of each event is the ratio of the area of the ap-
5. Two very busy men, A and B, who wish to confer, agree to appear at a designated place on a certain day, but no earlier than noon and no later than 12:15 p.m. If necessary, A will wait 6 minutes for B to arrive, while B will wait 9 minutes for A to arrive but neither can stay past 12:15 p.m. Express as a percent their chance of meeting.

Solution: Let A arrive $x$ minutes past noon and B at $y$ minutes past noon. Then $0 \leq x \leq 15$ and $0 \leq y \leq 15$. If A arrives first or both arrive simultaneously, then $y \geq x$ and $0 \leq y - x \leq 6$ if the men are to meet (Event I). Similarly if B arrives first then $x > y$ and $0 < x - y \leq 9$ (Event II). The probability of each event is the ratio of the area of the appropriate trapezoidal region to the area of the sample space ($15 \times 15$ square). Furthermore, since they are mutually exclusive, we may compute the desired probability by summing the probability of the two events. Thus, the chance of their meeting is

$$\frac{225 - \frac{1}{2}(81) - \frac{1}{2}(36)}{225} = 74\%$$