

EIGHTEENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

MATHEMATICAL ASSOCIATION OF AMERICA, MICHIGAN SECTION

PART II

December 4, 1974

INSTRUCTIONS

(to be read aloud to class by supervisor or proctor)

1. Record, in the upper righthand corner of this page, your six digit student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.
3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.
4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper righthand corner of each sheet.
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.
6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score
 1 2 3 4 5 Total

1. Let S be the sum of the 99 terms:

$$(\sqrt{1} + \sqrt{2})^{-1}, (\sqrt{2} + \sqrt{3})^{-1}, (\sqrt{3} + \sqrt{4})^{-1}, \dots, (\sqrt{99} + \sqrt{100})^{-1}.$$

Prove that S is an integer.

2. Determine all pairs of positive integers x and y for which $N = x^4 + 4y^4$ is a prime. (Your work should indicate why no other solutions are possible.)

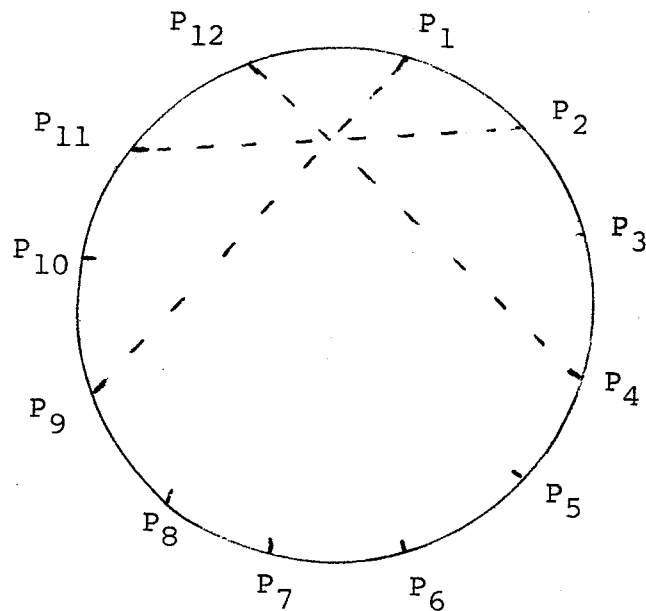
3. Let w, x, y, z be arbitrary positive real numbers. Prove each inequality:

$$(a) \quad xy \leq \left(\frac{x+y}{2}\right)^2$$

$$(b) \quad wxyz \leq \left(\frac{w+x+y+z}{4}\right)^4$$

$$(c) \quad xyz \leq \left(\frac{x+y+z}{3}\right)^3$$

4. Twelve points P_1, P_2, \dots, P_{12} are equally spaced on a circle, as shown. Prove that the chords $\overline{P_1P_9}$, $\overline{P_4P_{12}}$, and $\overline{P_2P_{11}}$ have a point in common.



5. Two very busy men, A and B, who wish to confer, agree to appear at a designated place on a certain day, but no earlier than noon and no later than 12:15 p.m. If necessary, A will wait 6 minutes for B to arrive, while B will wait 9 minutes for A to arrive but neither can stay past 12:15 p.m. Express as a percent their chance of meeting.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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