Student Number

_ _ _ _ _ _

SIXTEENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America, Michigan Colleges and Universities, Professional Organizations, and Industries.

PART II

December 6, 1972

INSTRUCTIONS
(to be read aloud to class by supervisor or proctor)

1. Record, in the upper righthand corner of this page, your six digit student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.

3. You are not expected to solve all problems completely. Look over all problems and work first on those which interest you the most.

4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper righthand corner of each sheet.

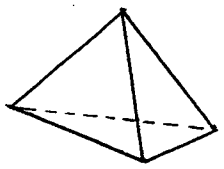
5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved. If you have difficulty understanding what is required in a given problem, note this on your answer sheet and attempt to make a non-trivial restatement of the problem. Then try to solve the restated problem.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a careful stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score					
1	2	3	4	5	Total

1. In a given tetrahedron the sum of the measures of the three face angles at each of the vertices is 180 degrees. Prove that all faces of the tetrahedron are congruent triangles.



2. The digital sum $D(n)$ of a positive integer n is defined recursively by:

$D(n) = n$ if $1 \leq n \leq 9$

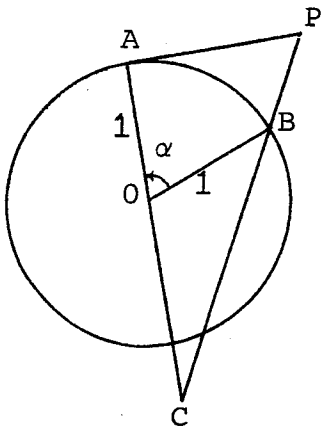
$D(n) = D(a_0 + a_1 + \ldots + a_m)$ if $n > 9$

where $a_0, a_1, \ldots, a_m$ are all the digits of n expressed in base ten. (For example, $D(989) = D(26) = D(8) = 8$.)

Prove that $D(n \times 1234) = D(n)$ for all positive integers n.

3. A right triangle has area A and perimeter P. Find the largest possible value for the positive constant k such that for every such triangle, $P^2 \geq kA$.

4. In the accompanying diagram, $\overline{AP}$ is tangent at A to a circle of radius 1 centered at 0. The segment $\overline{AP}$ is equal in length to the arc $\overset{\frown}{AB}$. Let C be the point of intersection of the lines A0 and PB. Determine the length of segment $\overline{AC}$ in terms of α, where α is the measure of $\angle$ A0B in radians.



5. Let $a_1 = a > 0$ and $a_2 = b > a$. Consider the sequence $\{a_1, a_2, a_3, \ldots\}$ of positive numbers defined by: $a_3 = \sqrt{a_1 a_2}$, $a_4 = \sqrt{a_2 a_3}$, ... , and in general, $a_n = \sqrt{a_{n-2} a_{n-1}}$ for $n \geq 3$. Develop a formula expressing a_n explicitly in terms of a, b and n, and determine $\lim_{n \to \infty} a_n$.

The following Michigan companies have made financial contributions to the Michigan Mathematics Prize Competition.

Hammond Machinery Builders, Inc., Kalamazoo

Michigan Bell Telephone Company, Detroit

Agencies submitting contributions to this competition after the printing of this examination will be recognized at the Annual Awards Banquet sponsored by Michigan Bell Telephone.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

DIRECTOR

R. A. Laing
Western Michigan University

OFFICERS OF THE MICHIGAN SECTION

Chairman

D. J. Lewis
The University of Michigan

Vice-Chairman

G. F. Feeman
Oakland University

Secretary-Treasurer

Y. Alavi
Western Michigan University

Governor

M. S. Klamkin
Ford Scientific Laboratory

EXAMINATION COMMITTEE

Chairman

P. J. Zwier
Calvin College

Stanislaw Leja
Western Michigan University

M. L. Tomber
Michigan State University

D. A. James
Wayne State University