School No. Student No.

_ _ _ _ _ _

FIFTEENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America, Michigan Colleges and Universities, Professional Organizations, and Industries.

PART 11

December 8, 1971

INSTRUCTIONS
(to be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, your school number and your student number. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part 11 consists of problems and proofs. You will be allowed 100 minutes for the five questions.

3. You are not expected to solve all questions completely. Look over all problems and work first on those which interest you the most.

4. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.

5. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.

6. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a careful stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

7. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

Score	1	2	3	4	5	Total

1. Prove that there is no interger n such that $n^2 + 1$ is divisible by 7.

2. Find all solutions of the system

$$x^2 - yz = 1$$

$$y^2 - xz = 2$$

$$z^2 - xy = 3$$

3. A <u>triangle with long legs</u> is an isoceles triangle in which the length of the two equal sides is greater than or equal to the length of the remaining side. What is the maximum number, n, of points in the plane with the property that every three of them form the vertices of a triangle with long legs?

Prove all assertions.

4. Prove that the area of a quadrilateral of sides a, b, c, d which can be inscribed in a circle and circumscribed about another circle is given by

$$A = \sqrt{abcd}$$

5. Prove that all of the squares of side length

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{n}, \ldots$$

can fit inside a square of side length 1 without overlap.

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this competition.

Kuhlman Electric Company, Birmingham

The Annual Awards Banquet is sponsored by Michigan Bell Telephone Company, Detroit.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

DIRECTOR

R. C. Hamelink
Michigan State University
East Lansing

OFFICERS OF THE MICHIGAN SECTION

Chairman:

W. T. Wechsler
Wayne State University

Vice-Chairman:

D. J. Lewis
University of Michigan

Secretary-Treasurer:

Y. Alavi
Western Michigan University

Governor:

M. S. Klamkin
Ford Scientific Laboratory

EXAMINATION COMMITTEE

Chairman:

T. F. Storer
University of Michigan

P. J. Zwier
Calvin College

Stanislaw Leja
Western Michigan University

M. L. Tomber
Michigan State University