IDENTIFICATION
NUMBER

_____

ELEVENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America, Michigan Colleges and Universities, Professional Organizations, and Industries

PART II

DECEMBER 6, 1967

INSTRUCTIONS
(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, the identification number from your questionnaire form. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the five questions.

3. Each problem is given equal weight and the total possible score on Part II is 50 points. The combined score on Part I and Part II will determine the final ranking of winners.

4. You are not expected to solve all the questions completely. Look over all problems and work first on those which interest you the most.

5. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.

6. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.

7. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

8. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

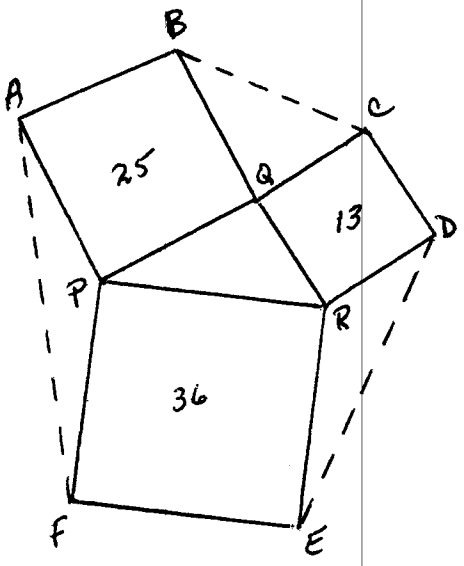
Problem No.1 _____	
Problem No.2 _____	
Problem No.3 _____	
Problem No.4 _____	
Problem No.5 _____	
TOTAL	

I. Consider the system of simultaneous equations

$$(x+y)(x+z) = a^2$$
$$(x+y)(y+z) = b^2$$
$$(x+z)(y+z) = c^2 \text{ , where } abc \neq 0.$$

Find all solutions (x,y,z) in terms of $a,b,$ and c.

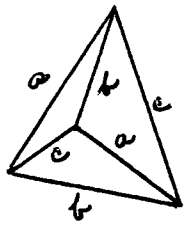
II.



Shown in the figure is a triangle PQR upon those sides squares of areas 13, 25, and 36 sq. units have been constructed. Find the area of the hexagon A B C D E F .

III. Suppose p,q, and r are positive integers no two of which have a common factor larger than 1. Suppose P,Q, and R are positive integers such that $\frac{P}{p} + \frac{Q}{q} + \frac{R}{r}$ is an integer. Prove that each of P/p, Q/q, and R/r is an integer.

IV. An isosceles tetrahedron is a tetrahedron in which opposite edges are congruent. Prove that all face angles of an isosceles tetrahedron are acute angles.



V. Suppose that p_1, p_2, p_3, and p_4 are the centers of four nonoverlapping circles of radius 1 in a plane and that p is any point in that plane. Prove that

$$\overline{p_1p}^2 + \overline{p_2p}^2 + \overline{p_3p}^2 + \overline{p_4p}^2 \geq 6 .$$

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this competition.

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