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# MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America,  
Michigan Colleges and Universities, Professional Organizations, and Industries

## PART II

DECEMBER 6, 1966

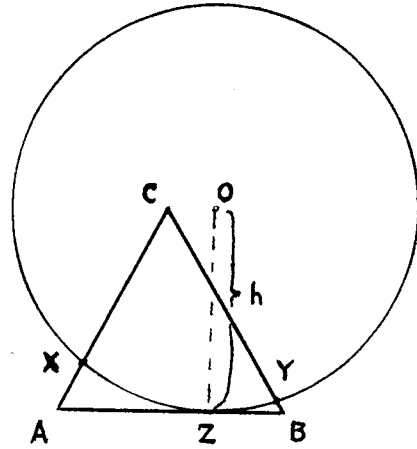
### INSTRUCTIONS

(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, the identification number from your questionnaire form. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the six questions.
3. Each problem is given equal weight and the total possible score on Part II is 60 points. The combined score on Part I and Part II will determine the final ranking of winners.
4. You are not expected to solve all the questions completely. Look over all problems and work first on those which interest you the most.
5. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.
6. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.
7. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
8. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

1. Each point in the interior and on the boundary of a square of side 2 inches is colored either red or blue. Prove that there exists at least one pair of points of the same color whose distance apart is not less than  $\sqrt{5}$  inches.

2.  $ABC$  is an equilateral triangle of altitude  $h$ . A circle with center  $O$  and radius  $h$  is tangent to side  $AB$  at  $Z$  and intersects side  $AC$  in point  $X$  and side  $BC$  in point  $Y$ . Prove that the circular arc  $\widehat{XZY}$  has measure  $60^\circ$ .



3. Find all of the real and complex solutions (if any exist) of the equation

$$x^7 + 7^7 = (x + 7)^7 .$$

4. The four points  $A$ ,  $B$ ,  $C$ , and  $D$  are not in the same plane. Given that the three angles, angle  $ABC$ , angle  $BCD$ , and angle  $CDA$ , are all right angles, prove that the fourth angle, angle  $DAB$ , of this skew quadrilateral is acute.

5. A, B, C and D are four positive whole numbers with the following properties:

(i) each is less than the sum of the other three, and (ii) each is a factor of the sum of the other three. Prove that at least two of the numbers must be equal. (An example of four such numbers:  $A = 4, B = 4, C = 2, D = 2.$ )

6.  $S$  is a set of six points and  $L$  is a set of straight line segments connecting certain pairs of points in  $S$  so that each point of  $S$  is connected with at least four of the other points. Let  $A$  and  $B$  denote two arbitrary points of  $S$ . Show that among the triangles having sides in  $L$  and vertices in  $S$  there are two with the properties:

- (i) The two triangles have no common vertex.
- (ii)  $A$  is a vertex of one of the triangles, and  $B$  is a vertex of the other.

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this year's competition:

Aeroquip Foundation, Jackson  
 Electro-Voice, Buchanan  
 Long Manufacturing, Detroit  
 Packaging Corporation, Filer City  
 Misco Precision Casting Company, Muskegon  
 Consumers Power Company, Jackson  
 The Michigan Council of Teachers of Mathematics  
 Burroughs Corporation, Detroit

The names of other companies, contributing to the scholarship fund during the next few months, will be reported in later announcements.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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Problem #1	_____
Problem #2	_____
Problem #3	_____
Problem #4	_____
Problem #5	_____
Problem #6	_____
TOTAL	