
MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America,
Michigan Colleges and Universities, Professional Organizations, and Industries

PART II

DECEMBER 14, 1965

INSTRUCTIONS

(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, the identification number from your questionnaire form. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.
2. Part II consists of problems and proofs. You will be allowed 100 minutes for the six questions.
3. Each problem is given equal weight and the total possible score on Part II is 60 points. The combined score on Part I and Part II will determine the final ranking of winners.
4. You are not expected to solve all the questions completely. Look over all problems and work first on those which interest you the most.
5. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.
6. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.
7. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.
8. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

1. For what integers x is it possible to find an integer y such that

$$x(x + 1)(x + 2)(x + 3) + 1 = y^2$$

2. Two tangents to a circle are parallel and touch the circle at points A and B , respectively. A tangent to the circle at any point X , other than A or B , meets the first tangent at Y and the second tangent at Z . Prove $AY \cdot BZ$ is independent of the position of X .

3. If a, b, c are positive real numbers, prove that

$$8abc \leq (b + c)(c + a)(a + b)$$

by first verifying the relation in the special case when $c = b$.

4. Solve the equation

$$\frac{x^2}{3} + \frac{48}{x^2} = 10\left(\frac{x}{3} - \frac{4}{x}\right)$$

5. Tom and Bill live on the same street. Each boy has a package to deliver to the other boy's house. The two boys start simultaneously from their own homes and meet 600 yards from Bill's house. The boys continue on their errand and they meet again 700 yards from Tom's house. How far apart do the boy's live?

6. A standard set of dominoes consists of 28 blocks of size 1 by 2. Each block contains two numbers from the set $0, 1, 2, \dots, 6$. We can denote the block containing 2 and 3 by $[2, 3]$, which is the same block as $[3, 2]$. The blocks $[0, 0], [1, 1], \dots, [6, 6]$ are in the set but there are no duplicate blocks.
- a) Show that it is possible to arrange the twenty-eight dominoes in a line, end-to-end, with adjacent ends matching, e. g., $\dots [3, 1][1, 1][1, 0] [0, 6] \dots$.
- b) Consider the set of dominoes which do not contain 0. Show that it is impossible to arrange this set in such a line.
- c) Generalize the problem and prove your generalization.

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this year's competition:

Aeroquip Corporation, Jackson
Burroughs Corporation, Detroit
Clark Equipment Company, Battle Creek
Consumers Power Company, Jackson
Electro-Voice, Incorporated, Buchanan
The Michigan Council of Teachers of Mathematics
Packaging Corporation of America, Filer City
Thompson Ramo Wooldridge, Incorporated, Warren
Whitman and Barnes, Plymouth

The names of other companies, contributing to the scholarship fund during the next few months, will be reported in later announcements.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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Problem #1	_____
Problem #2	_____
Problem #3	_____
Problem #4	_____
Problem #5	_____
Problem #6	_____
TOTAL	