IDENTIFICATION
NUMBER

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EIGHTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America,
Michigan Colleges and Universities, Professional Organizations, and Industries

PART II

DECEMBER 10, 1964

INSTRUCTIONS
(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, the identification number from your questionnaire form. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the six questions.

3. Each problem is given equal weight and the total possible score on Part II is 60 points. The combined score on Part I and Part II will determine the final ranking of winners.

4. You are not expected to solve all the questions completely. Look over all problems and work first on those which interest you the most.

5. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.

6. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.

7. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

8. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

1. The edges of a tetrahedron are all tangent to a sphere. Prove that the sum of the lengths of any pair of opposite edges equals the sum of the lengths of any other pair of opposite edges. (Two edges of a tetrahedron are said to be <u>opposite</u> if they do not have a vertex in common.)

2. Find the simplest formula possible for the product of the following 2n - 2 factors:

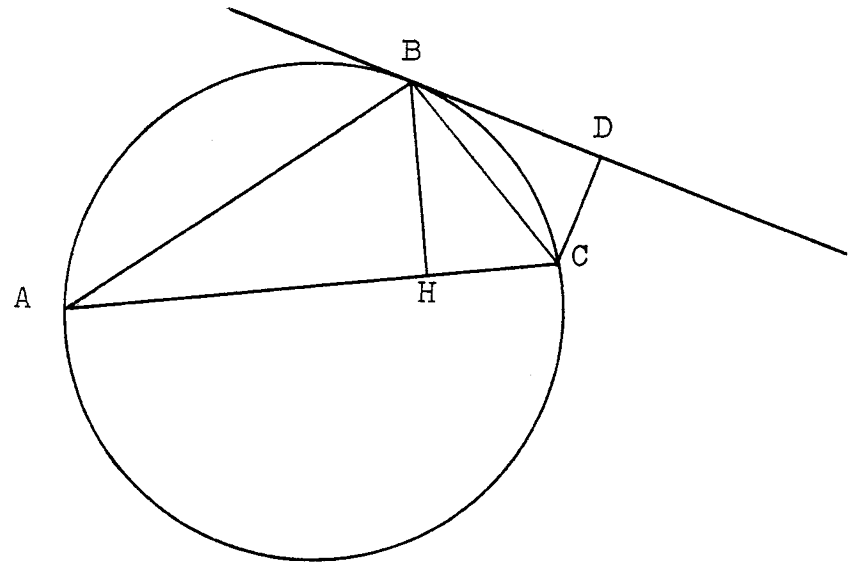
$(1 + \frac{1}{2}),\ (1 - \frac{1}{2}),\ (1 + \frac{1}{3}),\ (1 - \frac{1}{3}),\ \cdots,\ (1 + \frac{1}{n}),\ (1 - \frac{1}{n}).$

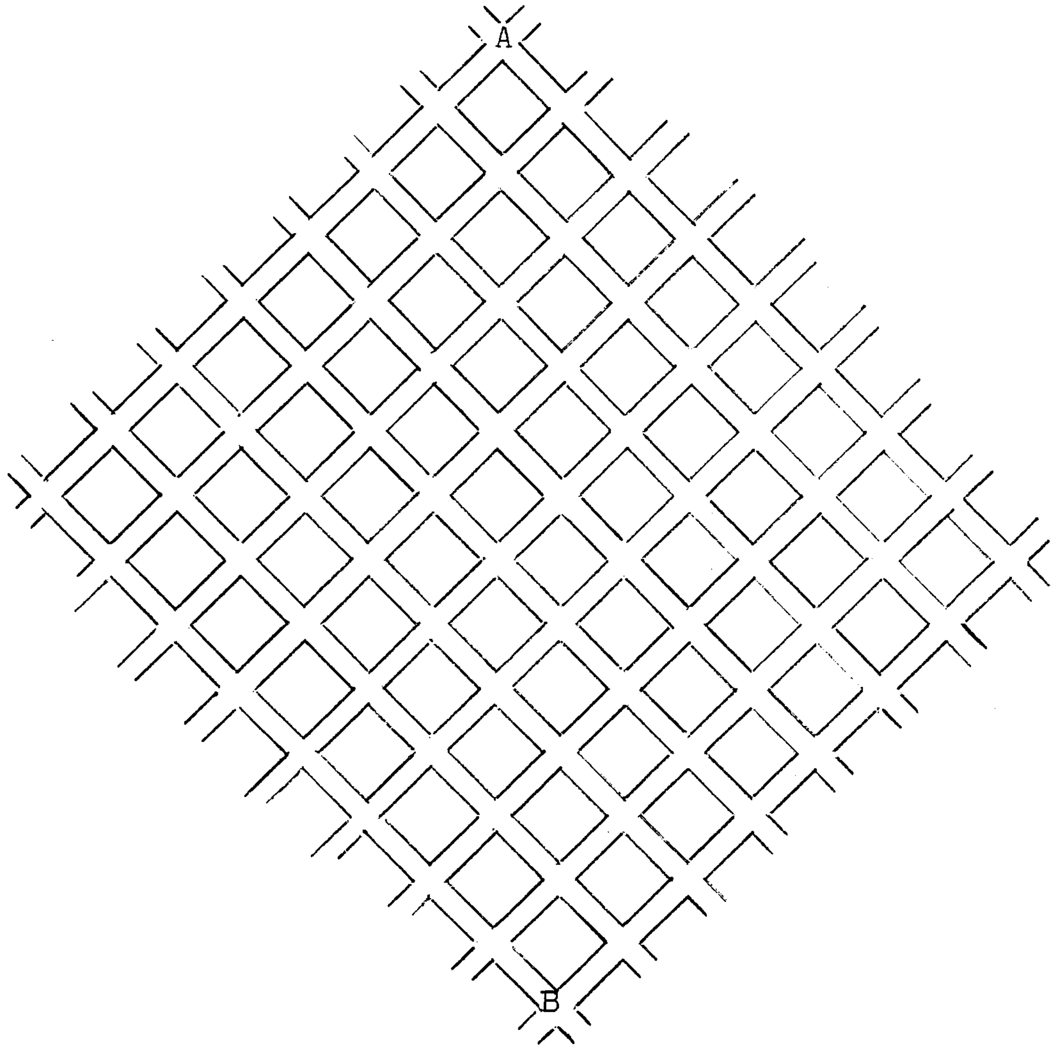
Prove that your formula is correct.

3. Solve:

$$\frac{(x+1)^2+1}{x+1} + \frac{(x+4)^2+4}{x+4} = \frac{(x+2)^2+2}{x+2} + \frac{(x+3)^2+3}{x+3}$$

4. Triangle ABC is inscribed in a circle, BD is tangent to this circle and CD is perpendicular to BD BH is the altitude from B to AC. Prove that the line DH is parallel to AB.



5. 

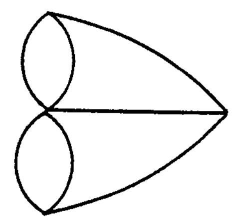
Consider the picture above as a section of a city street map. There are several paths from A to B, and if one always walks along the street, the shortest paths are 15 blocks in length. Find the number of paths of this length between A and B.

6. A finite graph is a set of points, called vertices, together with a set of arcs, called edges. Each edge connects two of the vertices (it is not necessary that every pair of vertices be connected by an edge).

The order of a vertex in a finite graph is the number of edges attached to that vertex.

Example

The figure at the right is a finite graph with 4 vertices and 7 edges. One vertex has order 5 and the other vertices order 3.



Define a finite graph to be heterogeneous if no two vertices have the same order.

Prove that no graph with two or more vertices is heterogeneous.

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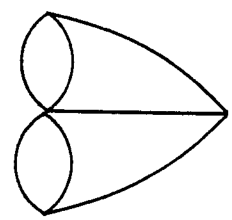

Define a finite graph to be heterogeneous if no two vertices have the same order.

Prove that no graph with two or more vertices is heterogeneous.

The following Michigan companies and professional organizations have made contributions to the scholarship fund for this year's competition:

- Aeroquip Corporation, Jackson
- Burroughs Corporation, Detroit
- Clark Equipment Company, Battle Creek
- Lear Siegler, Incorporated, Grand Rapids
- The Michigan Council of Teachers of Mathematics
- Packaging Corporation of America, Filer City
- Thompson Ramo Wooldridge, Incorporated, Warren
- Whitman and Barnes, Plymouth

It is anticipated that other contributions will be received during the next few months. The names of these other companies will be reported in later announcements.

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

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Problem #1	______
Problem #2	______
Problem #3	______
Problem #4	______
Problem #5	______
Problem #6	______
TOTAL	

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