IDENTIFICATION
NUMBER

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SEVENTH ANNUAL

MICHIGAN MATHEMATICS PRIZE COMPETITION

sponsored by

The Michigan Section of the Mathematical Association of America,
Michigan Colleges, Universities, Professional Organizations, and Industries

PART II

DECEMBER 12, 1963

INSTRUCTIONS
(To be read aloud to class by supervisor or proctor)

1. Record, in the upper lefthand corner of this page, the identification number from your questionnaire form. This is the only way to identify this test booklet with your name. Please do not write your name on the booklet.

2. Part II consists of problems and proofs. You will be allowed 100 minutes for the six questions.

3. Each problem is given equal weight and the total possible score on Part II is 60 points. The combined score on Part I and Part II will determine the final ranking of winners.

4. You are not expected to solve all the questions completely. Look over all problems and work first on those which interest you the most.

5. Each problem is on a different page. You should show most of your work on that page. If it is necessary to use additional paper for your answer, please indicate clearly your identification number and problem number in the upper lefthand corner of each sheet.

6. If you are unable to solve a particular problem, partial credit might be given for indicating a possible procedure or an example to illustrate the ideas involved.

7. You are advised to consider specializing or generalizing any problem where it seems appropriate. Sometimes an examination of special cases may generate ideas of how to attack the problem. On the other hand, a carefully stated generalization may justify additional credit provided you give an explanation of why the generalization might be true.

8. Your supervisor is not permitted to violate the rules by answering any questions. When the supervisor announces that the 100 minutes are up, please cease work immediately and insert all significant extra paper, including the questionnaire form, into the booklet. It is not necessary to return scratch paper on which routine numerical calculations were made.

1. Suppose $x \neq 1$ or 10 and logarithms are computed to the base 10. Define $y = 10^{\frac{1}{1 - \log x}}$ and $z = 10^{\frac{1}{1 - \log y}}$. Prove that $x = 10^{\frac{1}{1 - \log z}}$.

2. If n is an odd number and $x_1, x_2, x_3, \cdots, x_n$ is an arbitrary arrangement of the integers 1, 2, 3, $\cdots$, n, prove that the product $(x_1 - 1)(x_2 - 2)(x_3 - 3) \cdots (x_n - n)$ is an even number (possibly negative or zero).

3. Prove that $\frac{1 \cdot 3 \cdot 5 \cdots (2n - 1)}{2 \cdot 4 \cdot 6 \cdots 2n} < \sqrt{\frac{1}{2n + 1}}$ for all integers $n = 1,2,3,\cdots$.

4. Prove that if three angles of a convex polygon are each 60° then the polygon must be an equilateral triangle.

5. Find all solutions, real and complex, of

$$4\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) - 7 = 0$$

6. A man is $\frac{3}{8}$ of the way across a narrow railroad bridge when he hears a train approaching at 60 miles per hour. No matter which way he runs he can <u>just</u> escape being hit by the train. How fast can he run? Prove your assertion.

The following Michigan companies have made contributions to the scholarship fund for this year's competition:

Aeroquip Corporation, Jackson
Calumet and Hecla, Incorporated, Allen Park
Clark Equipment Company, Battle Creek
Lear Siegler, Incorporated, Grand Rapids
Misco Precision Casting Company, Whitehall
Whitman and Barnes, Plymouth

The Michigan Mathematics Prize Competition is an activity of the Michigan Section of the Mathematical Association of America.

Problem #1	____
Problem #2	____
Problem #3	____
Problem #4	____
Problem #5	____
Problem #6	____
TOTAL	____