FOURTH ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Sponsored by

Colleges, Universities, Professional and Industrial Organizations

In The State of Michigan

INSTRUCTIONS FOR PART II.

(To Be Read Aloud To Class by Supervisor or Proctor)

1. Part II is not a multiple choice test, but consists of problems and proofs. You will be allowed 60 minutes for five questions.

2. As In Part I, you are not expected to finish the five questions, so attempt to solve first those which interest you most.

3. On Part II, the Examiners will be more impressed by one question completely solved than by five questions each half solved. This is your opportunity to make a good impression, and the Examiners will take into account the way in which you attack a question and the way in which you explain your solution.

4. The special pencils need not be used on Part II. Make your preliminary calculations on blank paper supplied by your school, and write your solution in the space under the question in the examination booklet. If more space is needed, ask your supervisor for extra paper.

5. As In Part I, stop when your supervisor announces that the sixty minutes are up. As before, your supervisor is not permitted to violate the rules by answering any questions.

NAME____________________ HIGH SCHOOL GRADE______

HIGH SCHOOL____________________ CITY____________________

HIGH SCHOOL NUMBER____________

COLLEGES AND UNIVERSITIES TO WHICH ADMISSION APPLICATIONS

HAVE BEEN SUBMITTED______________________________

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SCHOLARSHIPS APPLIED FOR________________________

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March 2, 1961

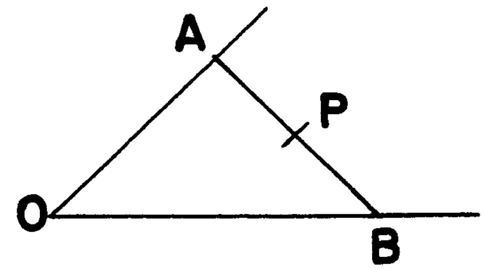
Part II

1. If x, y, z are required to be non-negative whole numbers, find all solutions to the pair of equations

$$x + y + z = 40$$
$$2x + 4y + 17z = 301.$$

Part II

2. Let P be a point lying between the sides of an acute angle whose vertex is O. Let A, B be the intersections of a line passing through P with the sides of the angle. Prove that the triangle AOB has minimum area when P bisects the line segment AB.



Part II

3. Find all values of x for which

$$|3x - 2| + |3x + 1| = 3.$$

Part II

4. Prove that $x^2 + y^2 + z^2$ cannot be factored in the form

$$(ax + by + cz)\,(dx + ey + fz),$$

a, b, c, d, e, f real.

Part II

5. Let $f(x)$ be a continuous function for all real values of x such that $f(a) \leq f(b)$ whenever $a \leq b$. Prove that, for every real number r, the equation

$$x + f(x) = r$$

has exactly one solution.