

PART II

FIRST ANNUAL MICHIGAN MATHEMATICS PRIZE COMPETITION

Sponsored by

Colleges, Universities, Professional and Industrial Organizations

In The State of Michigan

INSTRUCTIONS FOR PART II.

(To Be Read Aloud To Class By Supervisor or Proctor)

1. Part II is not a multiple choice test, but consists of problems and proofs. You will be allowed 60 minutes for five questions.
2. As in Part I, you are not expected to finish the five questions, so attempt to solve first those which interest you most.
3. On Part II, the Examiners will be more impressed by one question completely solved than by five questions each half solved. This is your opportunity to make a good impression, and the Examiners will take into account the way in which you attack a question and the way in which you explain your solution.
4. The special pencils need not be used on Part II. Make your preliminary calculations on blank paper supplied by your school, and write your solution in the space under the question in the examination booklet. If more space is needed, ask your supervisor for extra paper.
5. As in Part I, stop when your supervisor announces that the sixty minutes are up. As before, your supervisor is not permitted to violate the rules by answering any questions.

NAME _____

YEAR _____

HIGH SCHOOL _____

CITY _____

HIGH SCHOOL NUMBER _____

March 27, 1958

PART II

1. Show that $9x + 5y$ is a multiple of 17 whenever $2x + 3y$ is a multiple of 17.

2. Express the five distinct fifth roots of 1 in terms of radicals.

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3. Prove that the three perpendiculars dropped to the three sides of an equilateral triangle from any point inside the triangle have a constant sum.

4. Find the volume of a sphere which circumscribes a regular tetrahedron of edge a .

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5. For any integer n greater than 1, show that $n^2 - 2n + 1$ is a factor of $n^{n-1} - 1$.